



**British  
Geological Survey**  
NATIONAL ENVIRONMENT RESEARCH COUNCIL

# Gateway to the Earth

## Updated Centre of Mass Correction Tables for LAGEOS, Etalon, LARES, Starlette and Ajisai

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# Context

SLR and VLBI scale difference ITRF2014 (1.37 ppb)

Allowing in the orbit solutions for the presence of observational errors reduces this difference by ~50%

Identifying the actual error sources is very hard:

- Centre of mass corrections?
- Timing devices?
- Site surveys?
- Operational inconsistencies?
- Modelling deficiencies?
- ...other?

# Context

SLR measurements are made to the reflecting surfaces of the satellites: an offset to their centre of mass (CoM) is required to solve the equations of motion

Time of flight measurements are only as good as the CoM values applied (among other things)

Station heights estimated from SLR will absorb errors in the observations by a ratio of ~1:1

# Aims

Reassess centre of mass (CoM) models used by ILRS analysts for ITRF2014

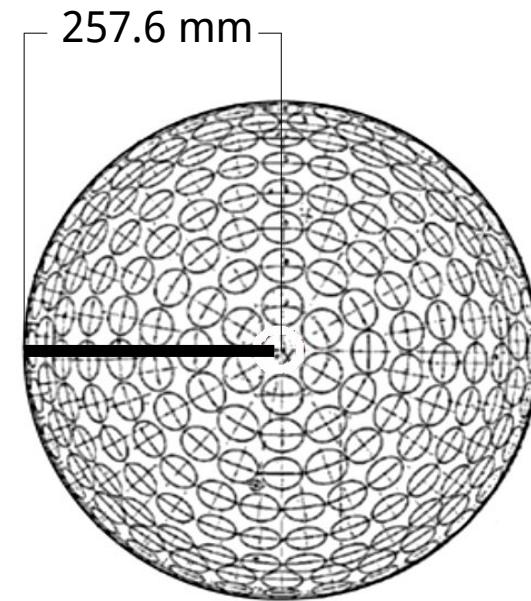
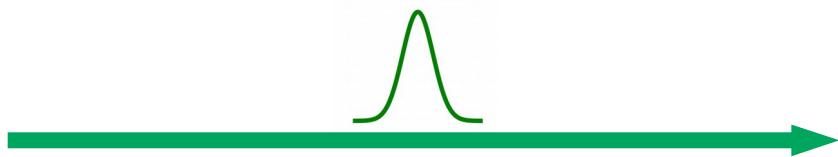
Improve current standards incorporating effects previously only approximated

Recompute everything from scratch using the latest data available

Assess the impact on the overall errors estimated in the orbital solutions

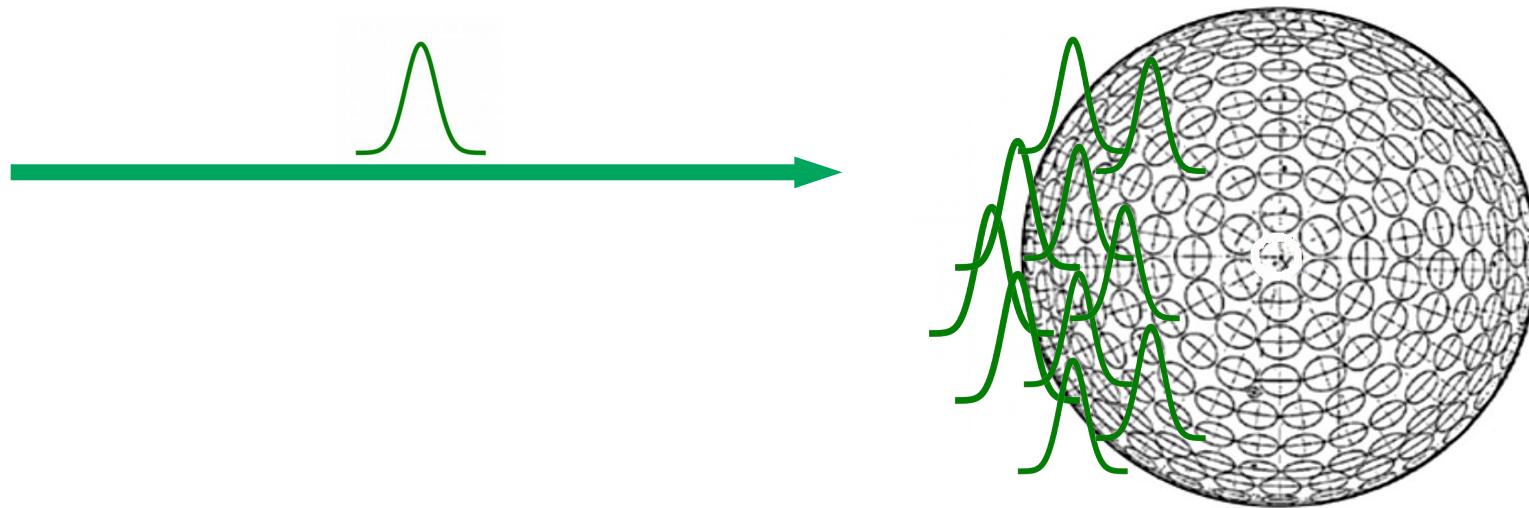
If we had perfect CoM values, estimated range errors could be transferred to other targets

**Question:** Why don't you just read the technical drawings?



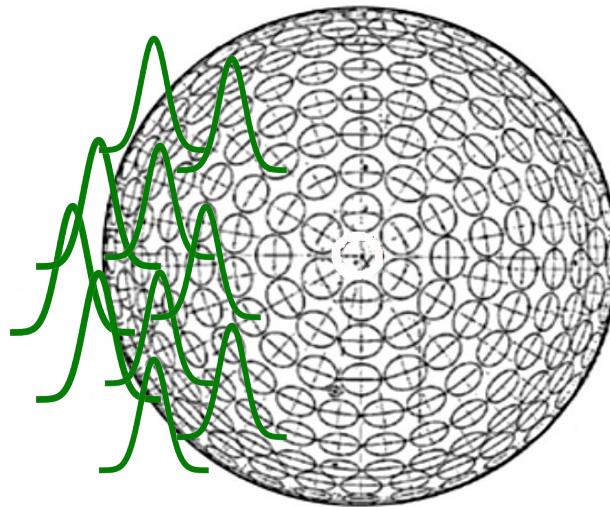
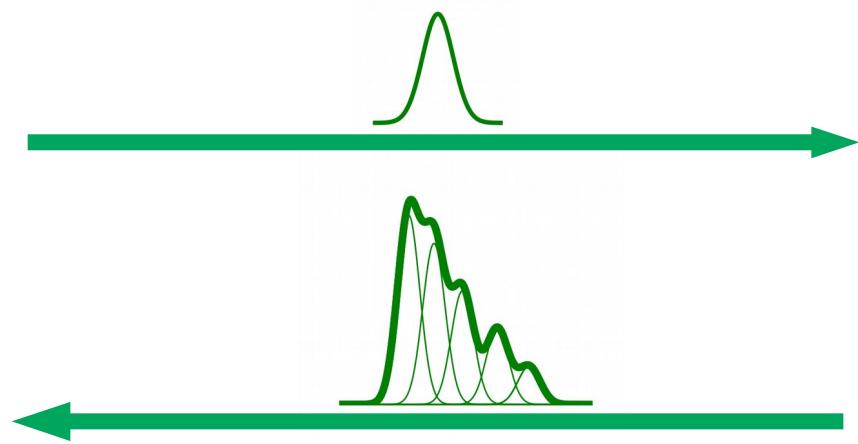
LAGEOS

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LAGEOS

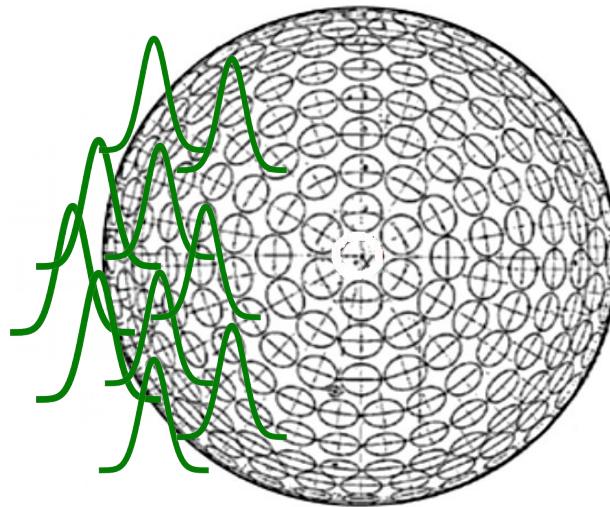
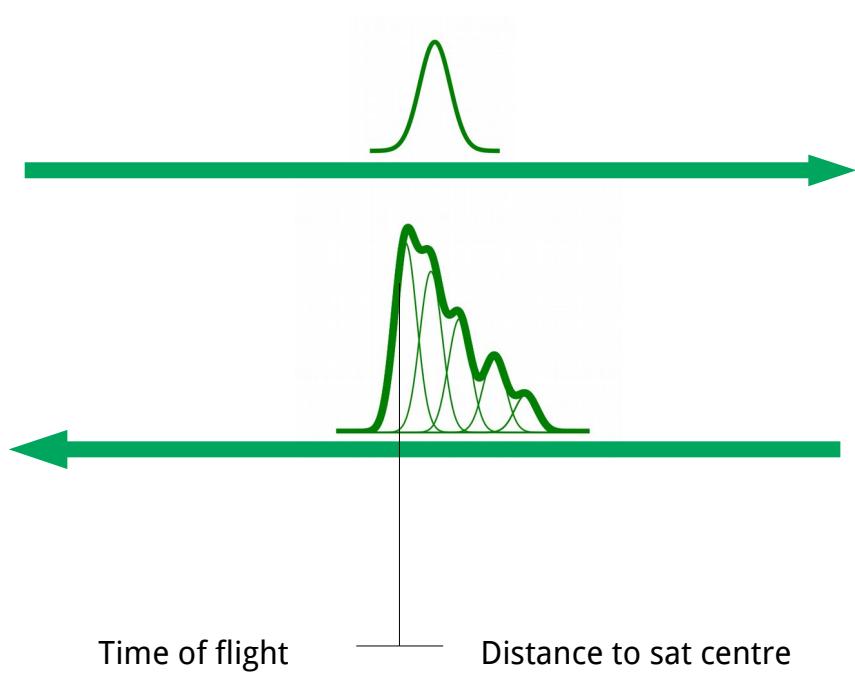
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LAGEOS

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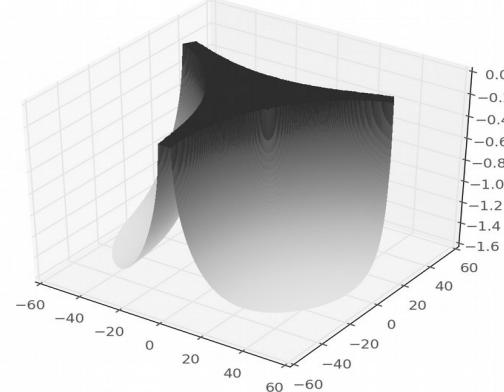
**Answer:** Target signature effects



LAGEOS

# CoM modelling steps

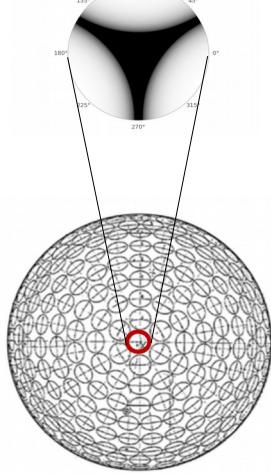
1. Compute ideal **optical response** of laser retroreflector arrays (LRA)
2. Determine **best fit** response using empirical data from distributions of single-photon detections
3. Compute CoM values using **system specifications**



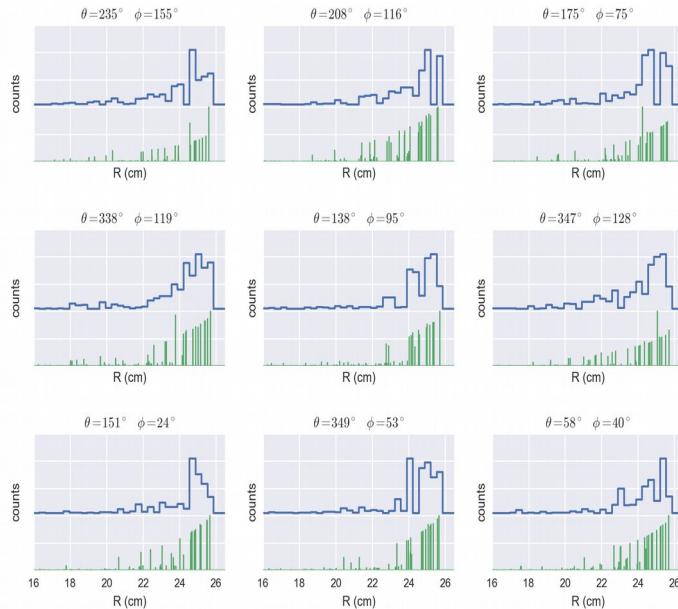
**Otsubo & Appleby**, System dependent CoM corrections for spherical satellites, 2003

# 1. Optical response function

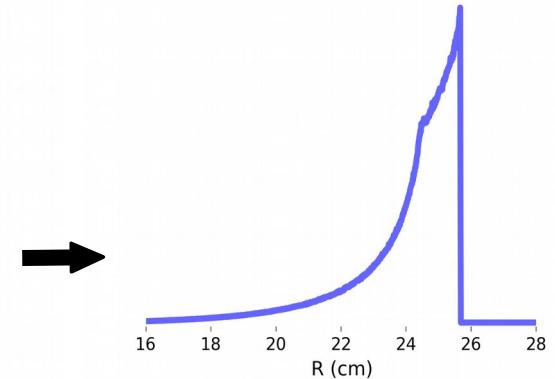
Reflectivity map



Response at arbitrary orientations



Average over 250K orientations



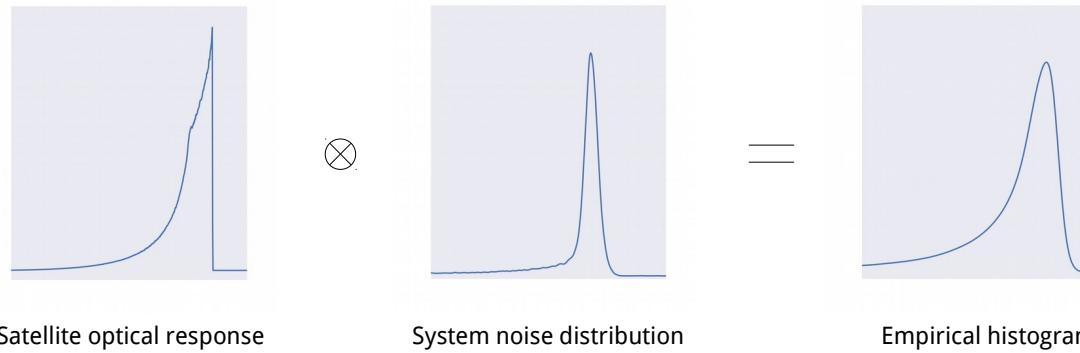
(family of functions)

## 2. Empirical fit

Accumulate single-photon detection data to obtain empirical distributions

We stacked full rate data from Herstmonceux station (2015-2018), selected, filtered and aligned

- ~ 15M LAGEOS obs.
- ~ 9.5M LAGEOS-2 obs.
- ~ 10.3M LARES obs.
- ~ 1.0M Etalon-1 + Etalon-2 obs
- ~ 4.5M Starlette obs.
- ~ 5.3M Ajisai obs.



What theoretical function fits the data best?

### **3. CoM computation**

Perform computation for all known system configurations

Input data: hardware parameters characterising system behaviour, average return rates and optical response functions

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#### **Single-photon ranging**

An analytical expression is available to compute distribution of returns

Solve numerically using calibration data provided by some stations (estimate from relevant system parameters otherwise)

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Solve numerically using calibration data provided by some stations (estimate from relevant system parameters otherwise)

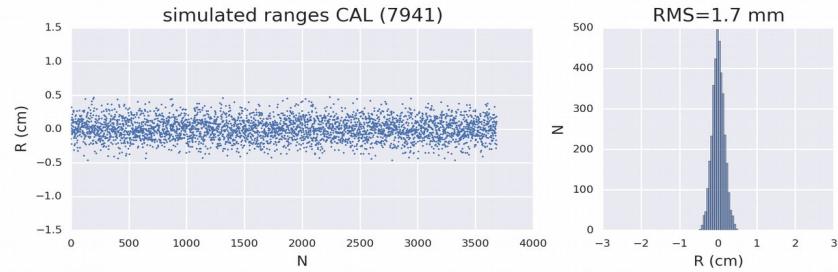
#### **Multi-photon ranging**

Monte Carlo numerical simulation of simplified, ideal detection process

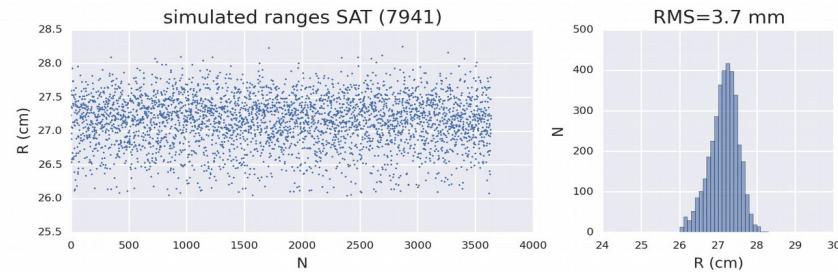
Dependent on some difficult-to-validate assumptions

### 3. CoM computation

CAL  
simulation

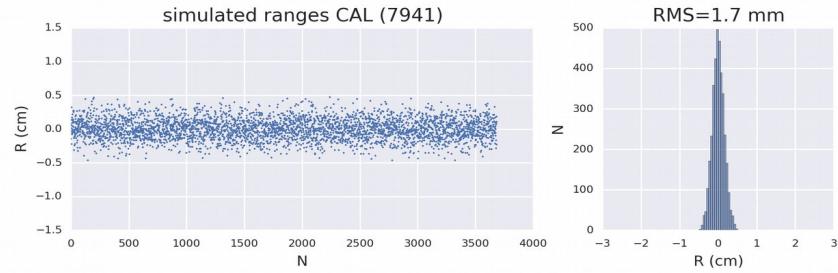


SAT  
simulation

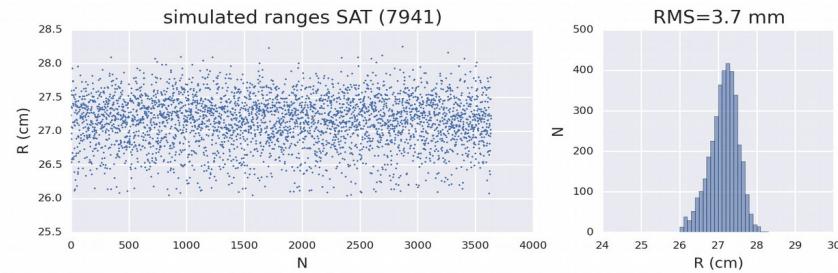


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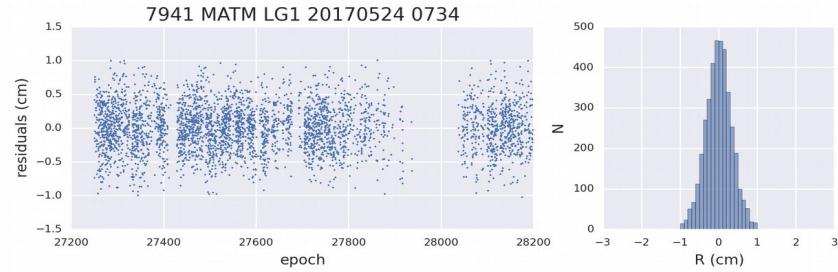
CAL  
simulation



SAT  
simulation

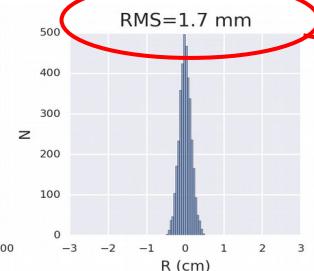
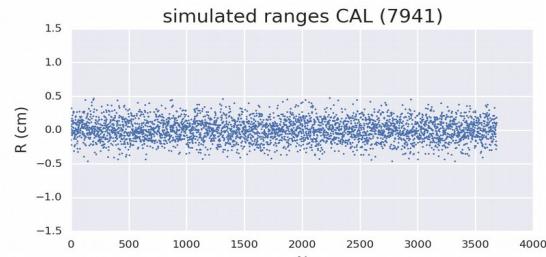


SAT  
empirical

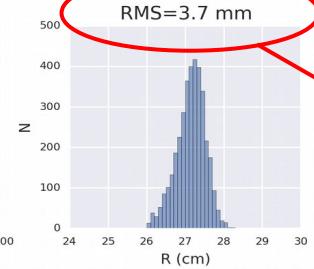
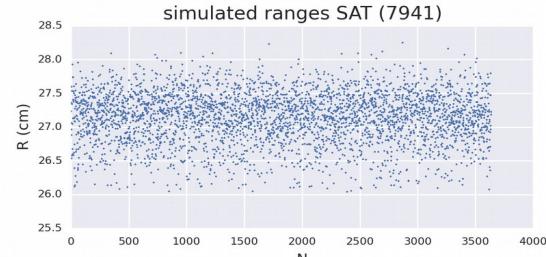


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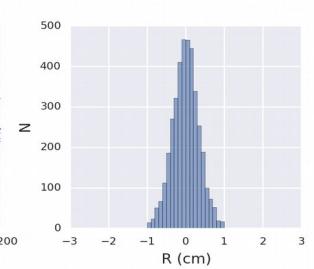
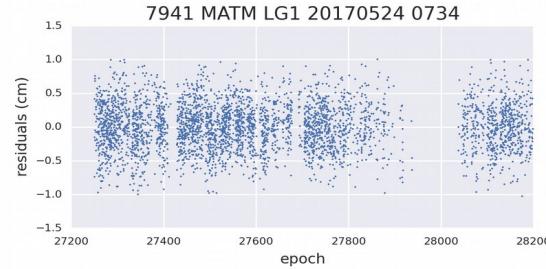
CAL  
simulation



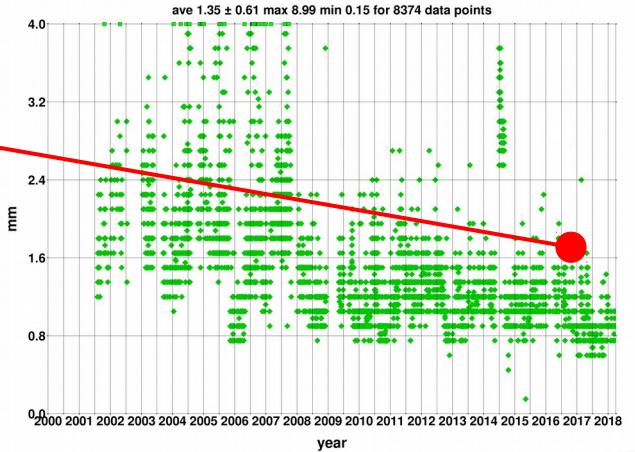
SAT  
simulation



SAT  
empirical

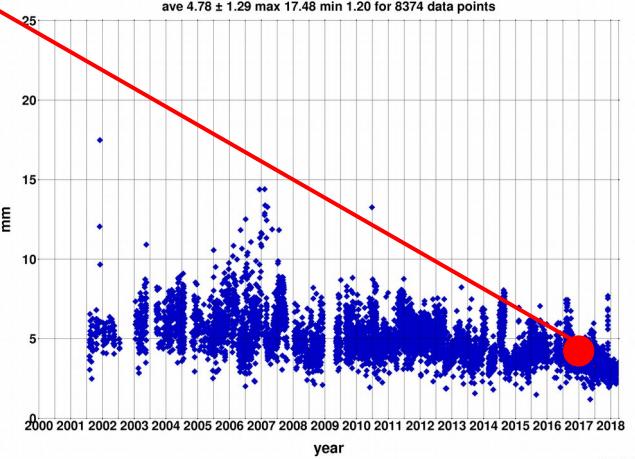


Matera, Italy (MLRO) 7941  
pass LAGEOS calibration rms



CAL RMS  
consistency?

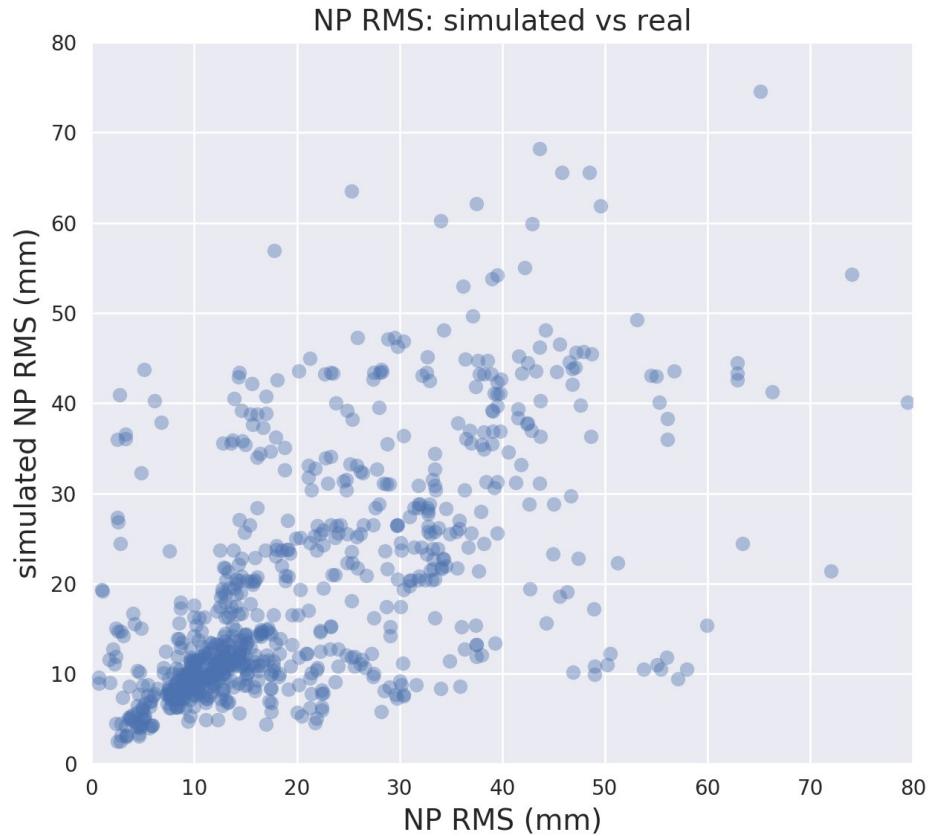
Matera, Italy (MLRO) 7941  
pass average LAGEOS normal point rms



SAT RMS  
consistency?

# Model validation/performance

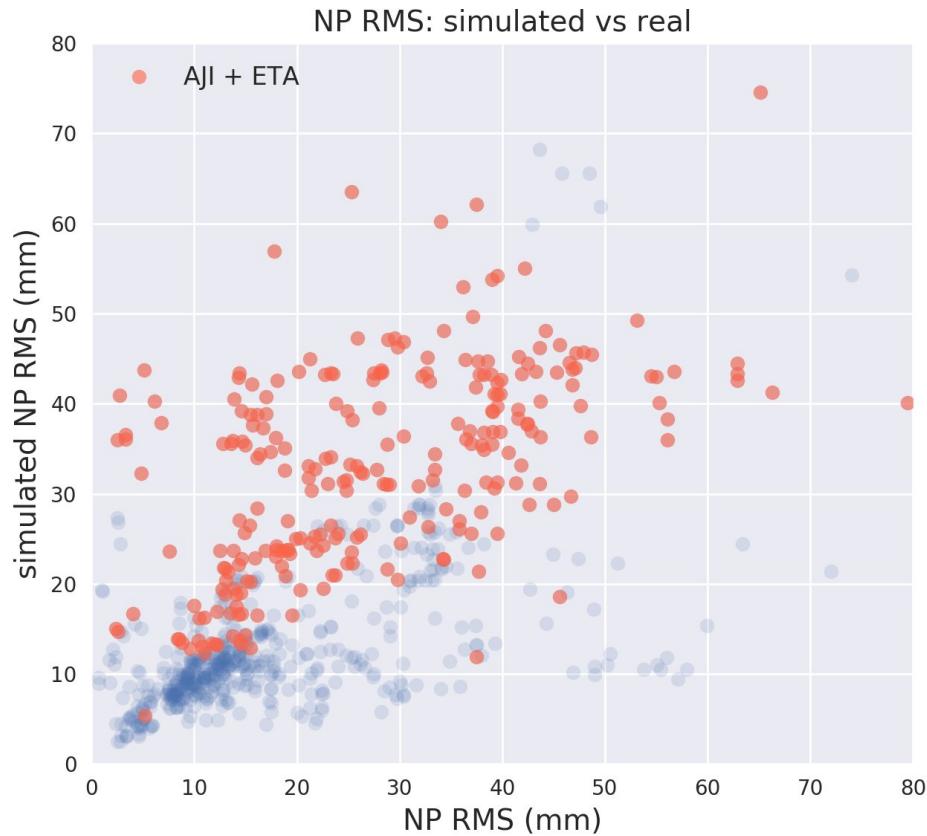
Comparison of predicted NP RMS with actual NP RMS provides a measure of model performance



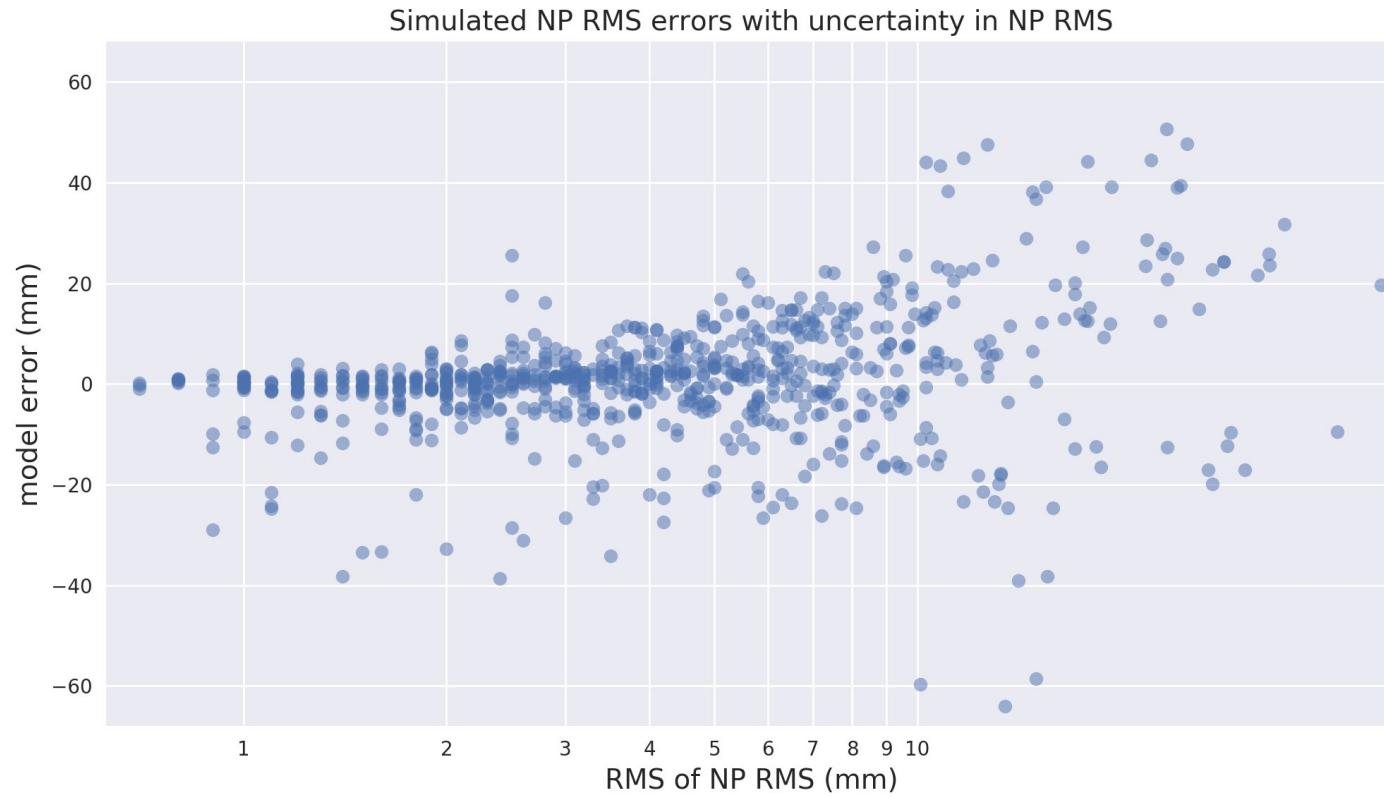
# Model validation/performance

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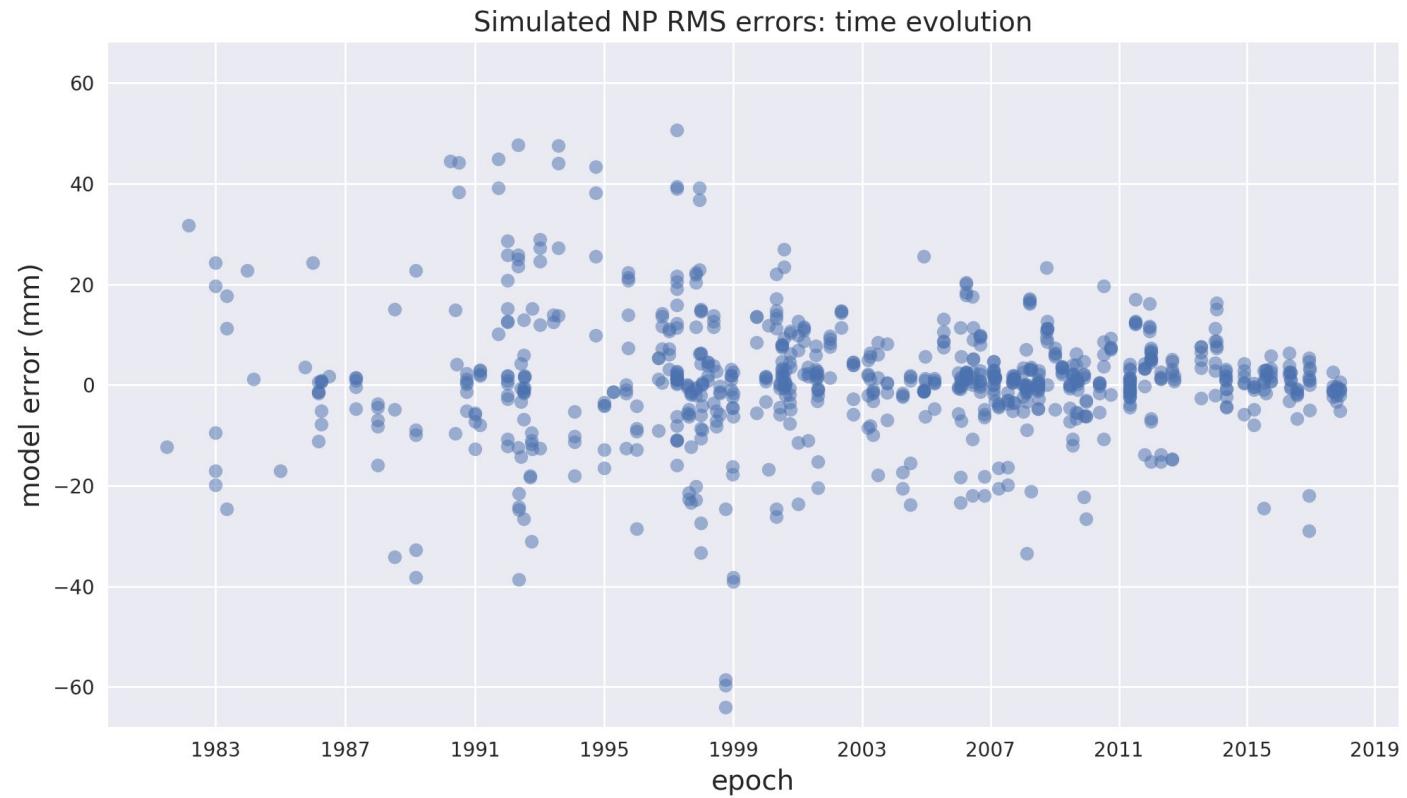
Bigger targets (AJI, ETA) are harder to model



# Model validation/performance

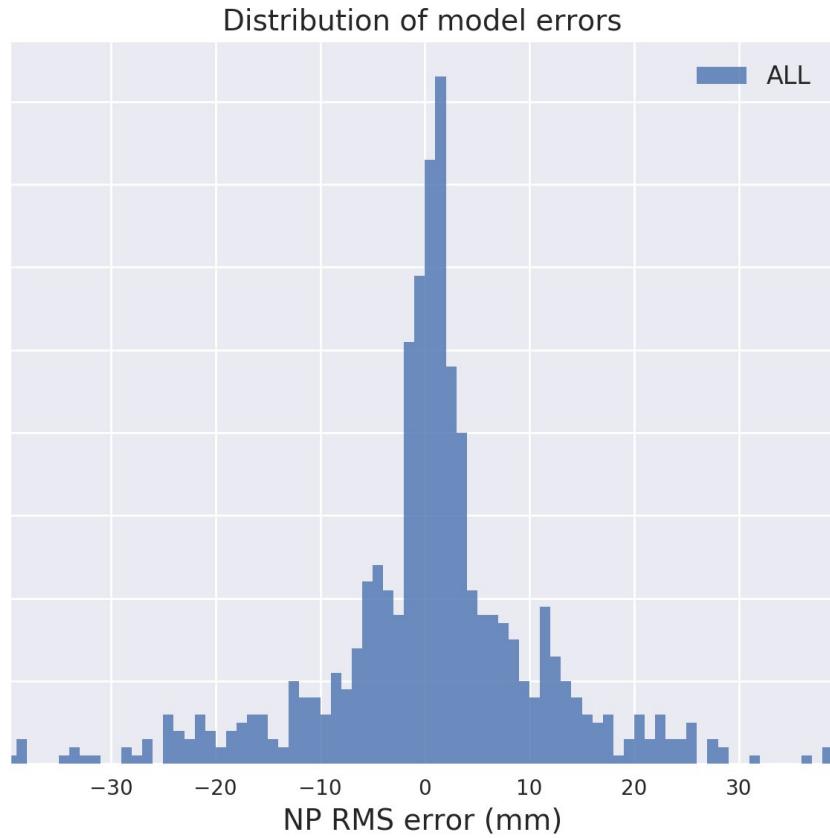


# Model validation/performance



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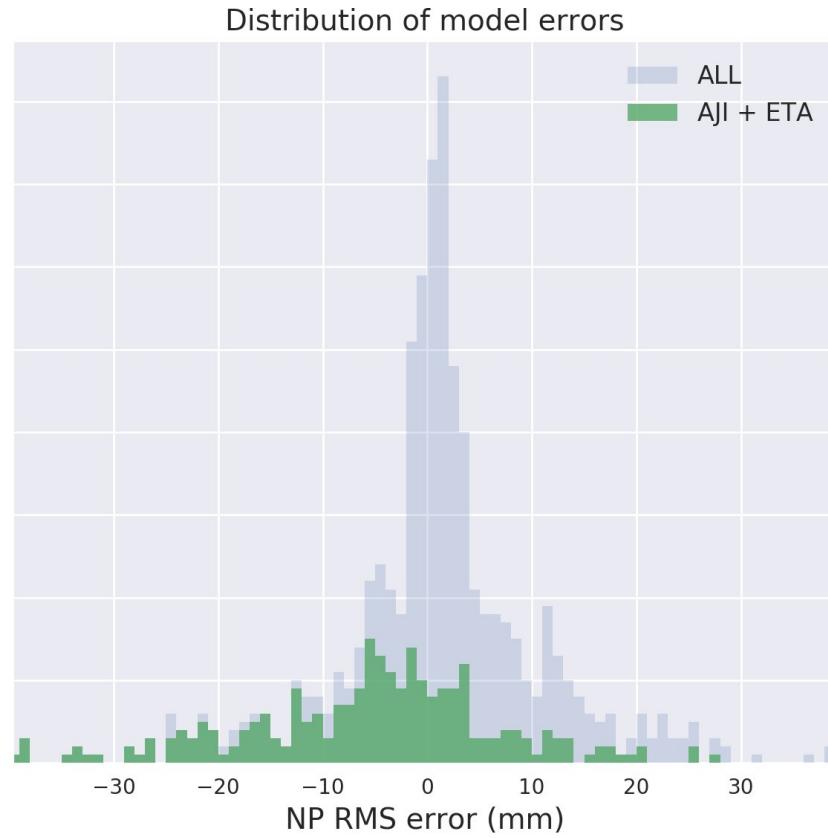
Despite expected caveats models performs reasonably well most of the time



# Model validation/performance

Despite expected caveats models performs reasonably well most of the time

Bigger targets = poorer predictions

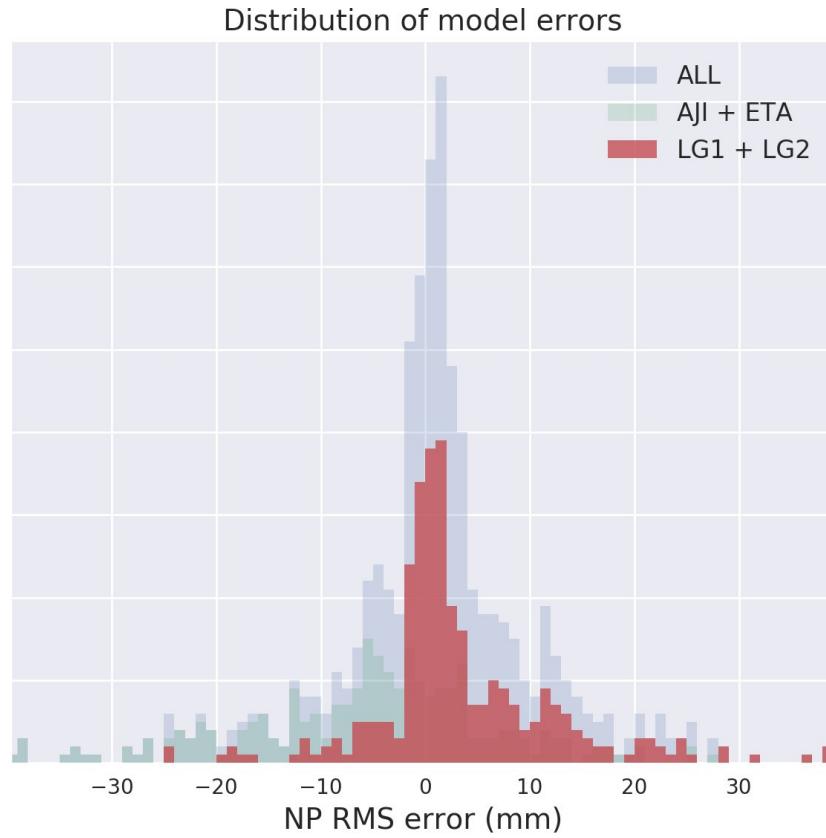


# Model validation/performance

Despite expected caveats models performs reasonably well most of the time

Bigger targets = poorer predictions

Most systems well predicted for smaller targets

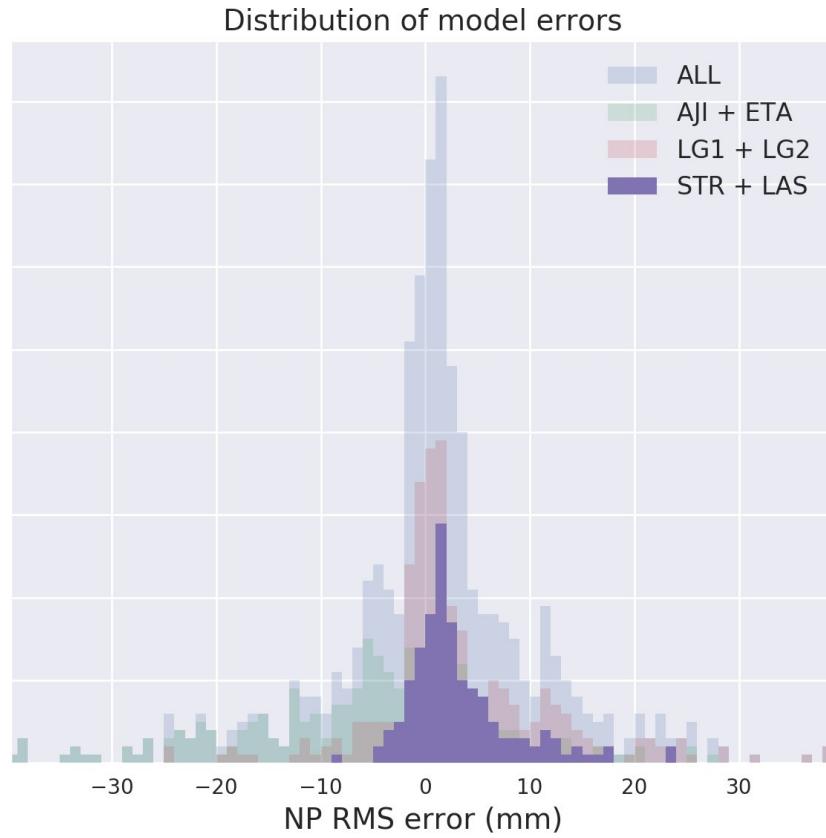


# Model validation/performance

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# Results

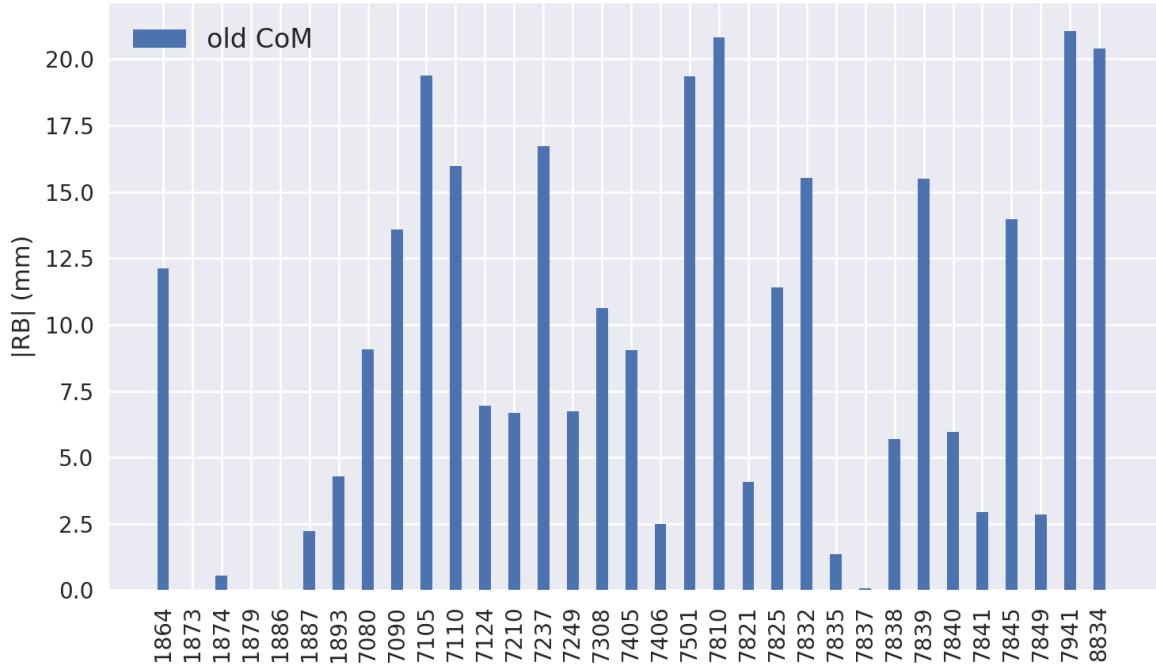
Centre of mass values computed for all stations of the network for 6 spherical satellites

We made comparisons of the estimated range errors obtained with the old and new CoM values

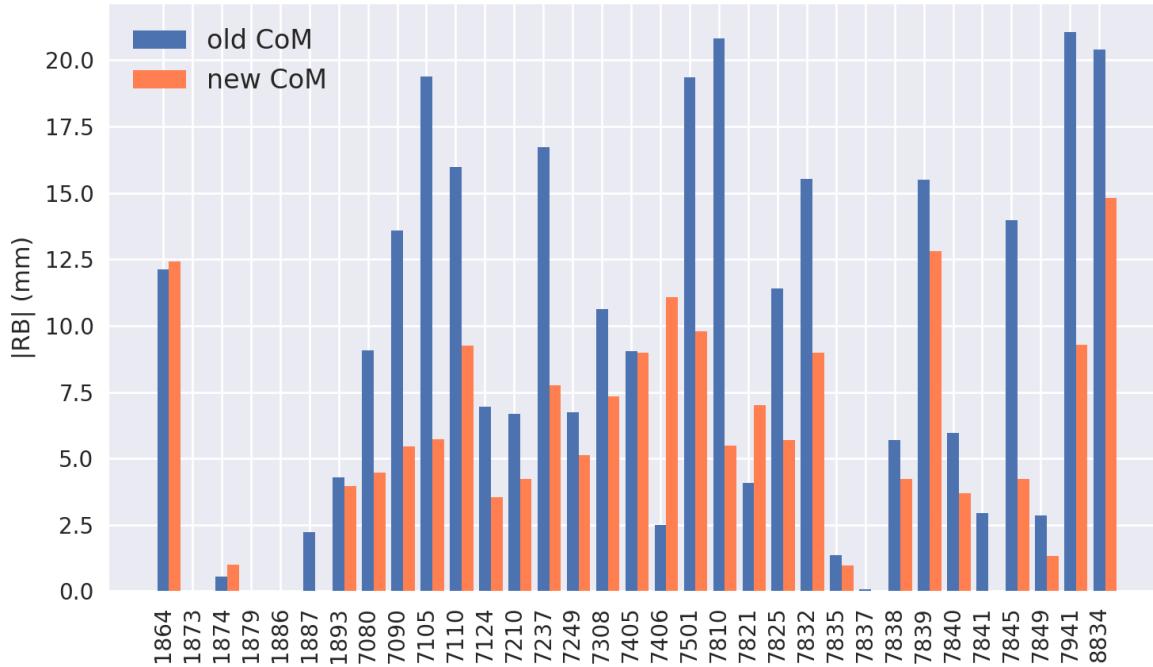
BUT: impossible to separate between range errors and CoM mismodelling

Assessed effect on station heights/frame scale

### ETALON mean $|RB|$ 2000-2018



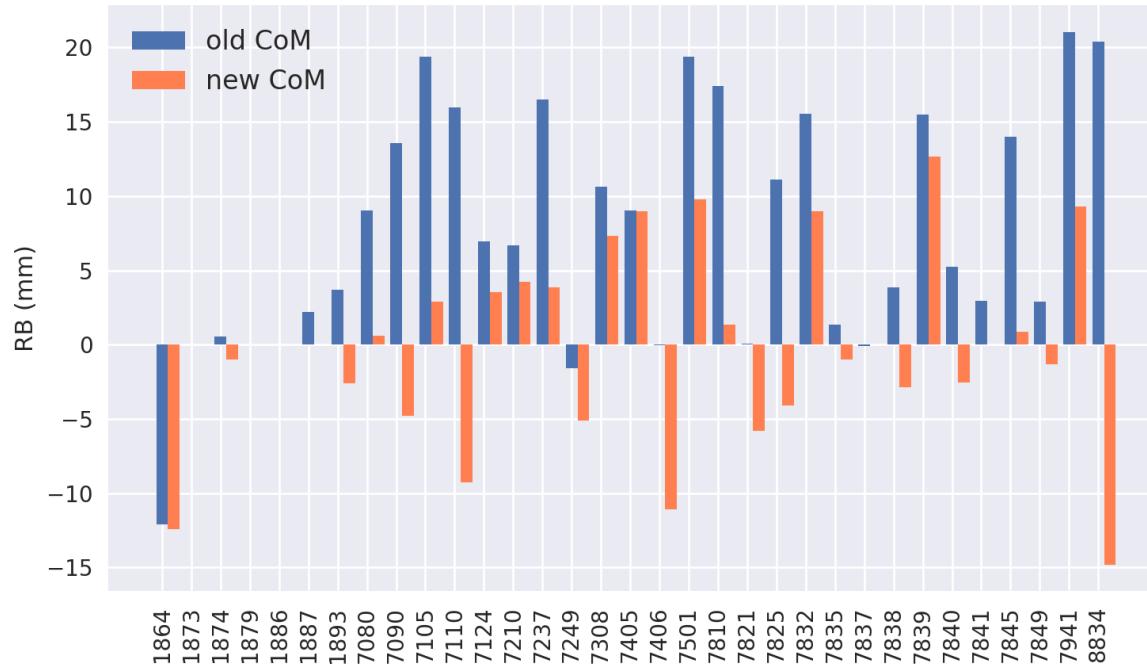
### ETALON mean $|RB|$ 2000-2018



For Etalon, test CoM values remove about 1 cm biases from several stations

Very few stations see an increase in RB

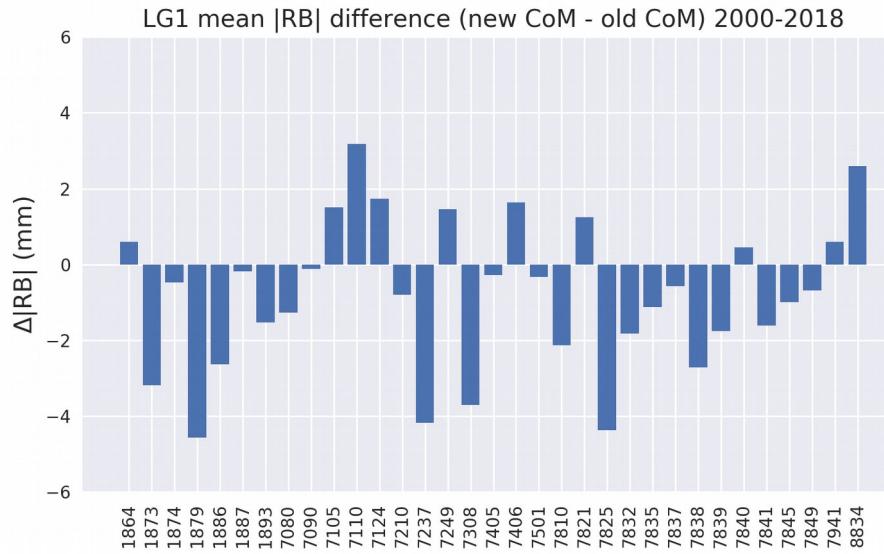
### ETALON mean RB 2000-2018



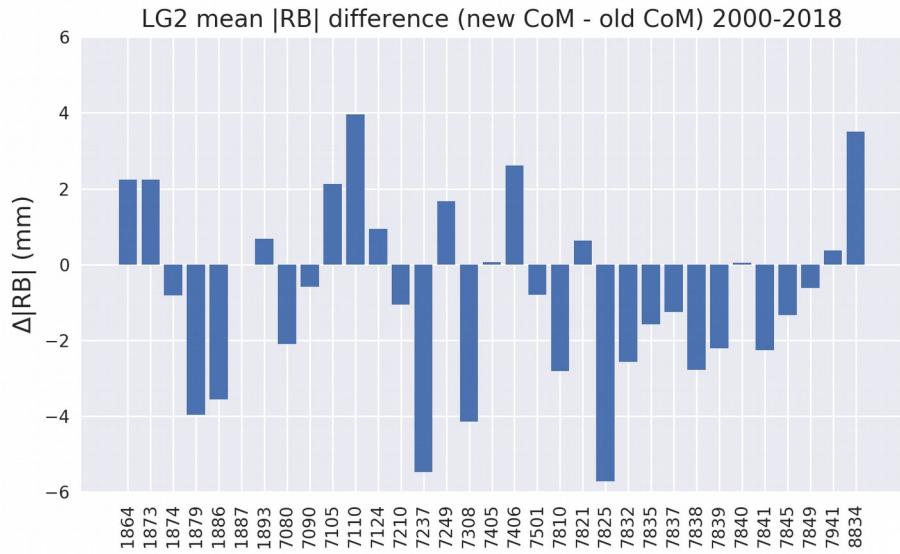
Actual differences are greater when considering the sign of the estimated biases

New CoM values remove to a large extent the predominant positive bias across the network

## LAGEOS



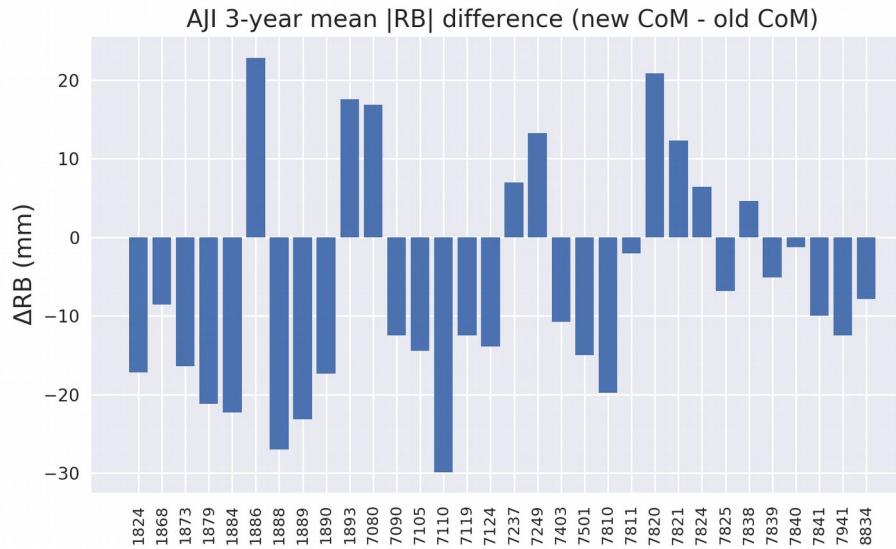
## LAGEOS-2



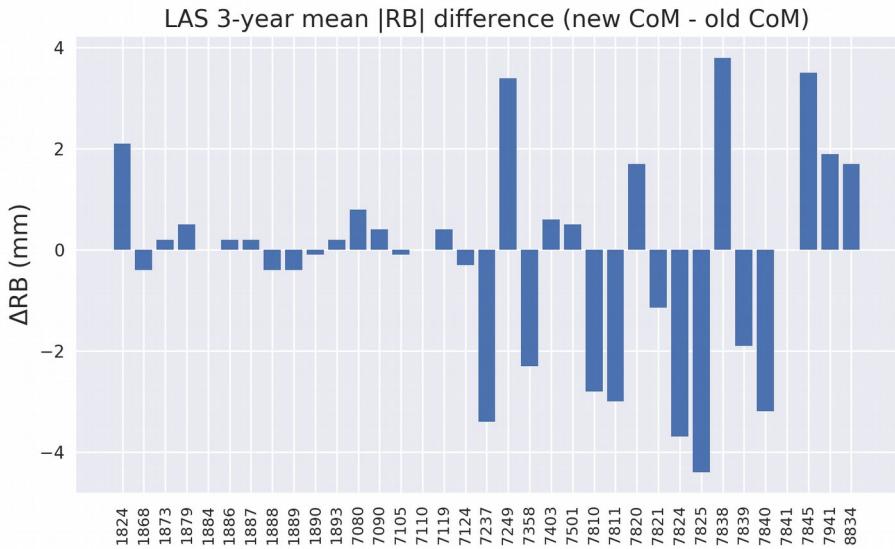
Negative is "good": RB "removed" from stations

Positive is "bad": RB "added" to stations

AJISAI

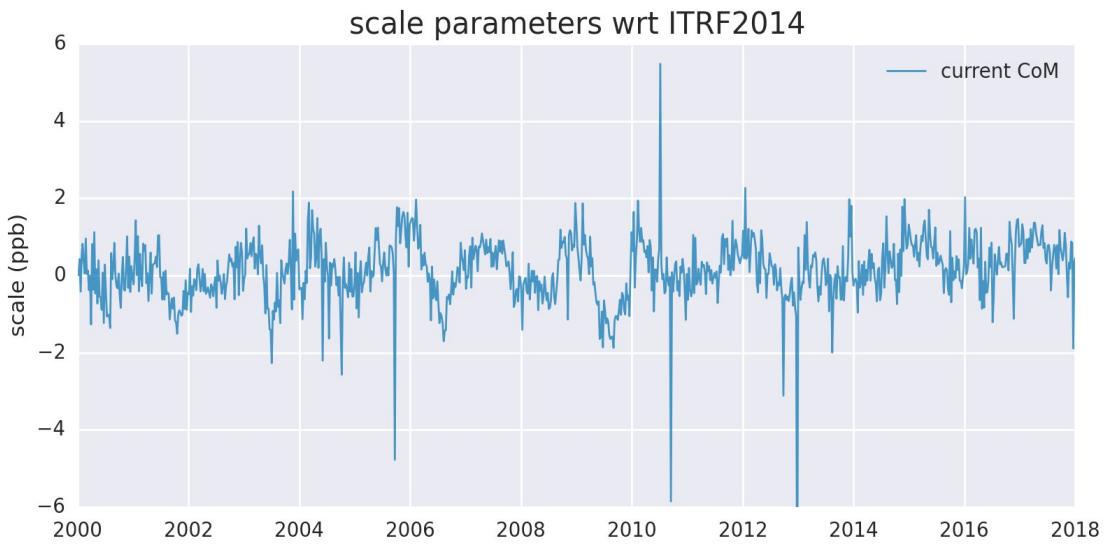


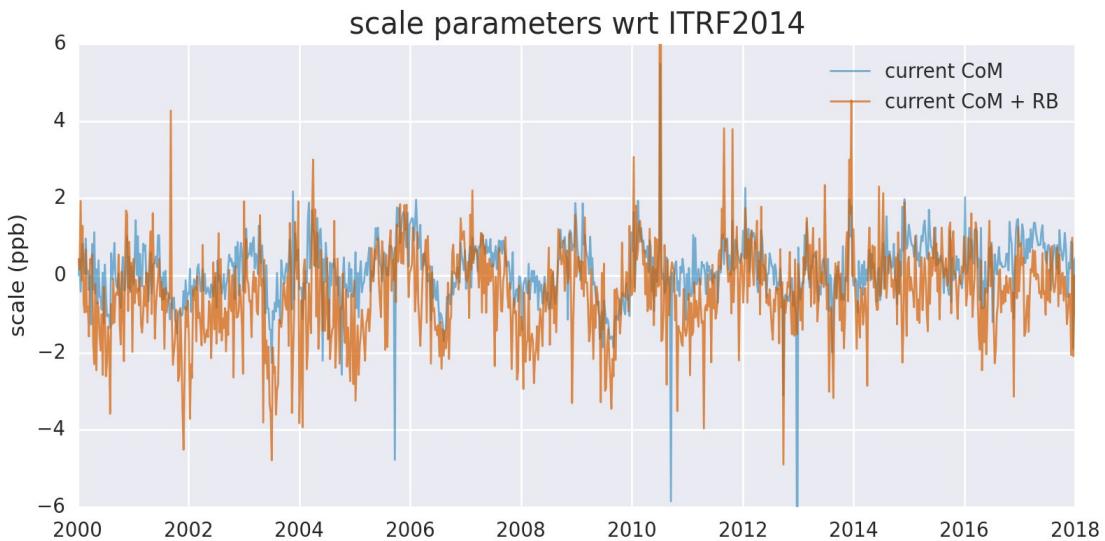
LARES

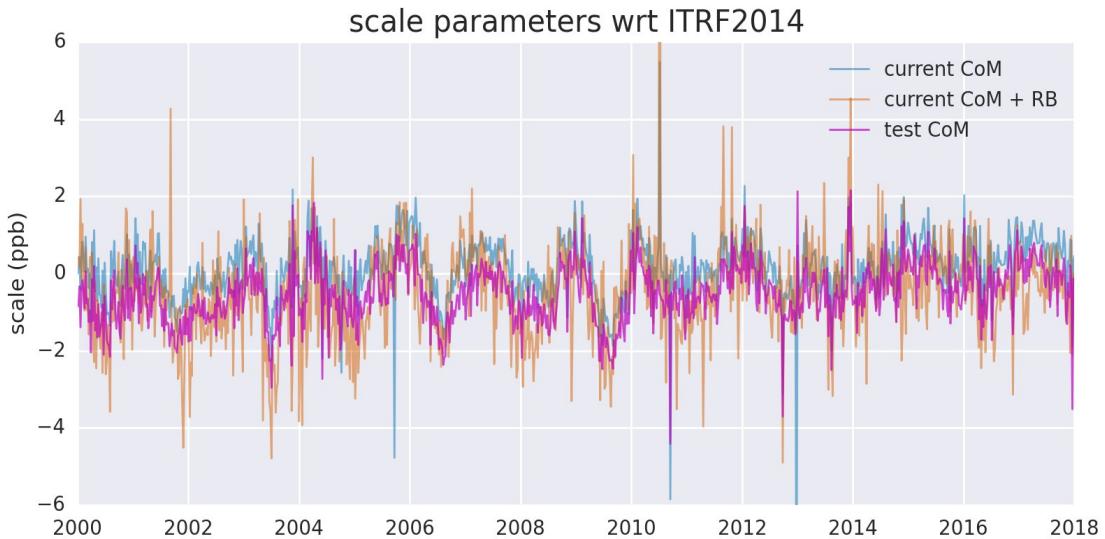


Negative is “good”: RB “removed” from stations

Positive is “bad”: RB “added” to stations







Similar average scale change when estimating RB and when using test CoM values: ~0.6 ppb

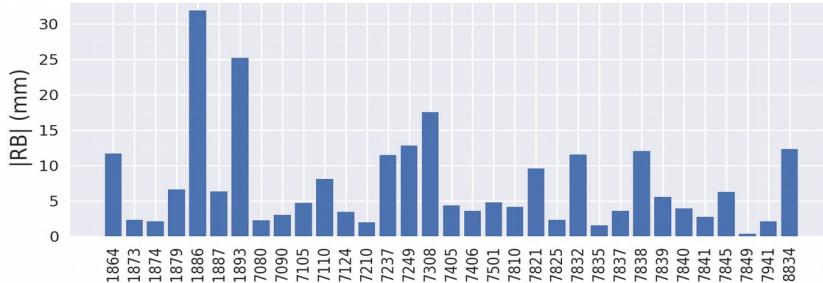
Or in other words: both solution types have **increased** station heights

Agreement between the scales realised by SLR and VLBI is improved

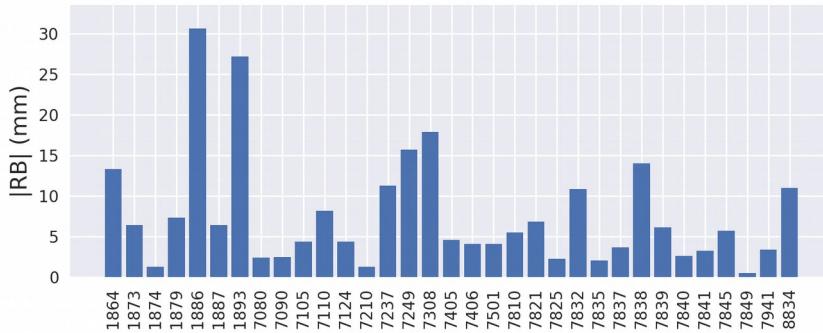
Have we solved the RB problems...?

|RB| remaining using new CoM values

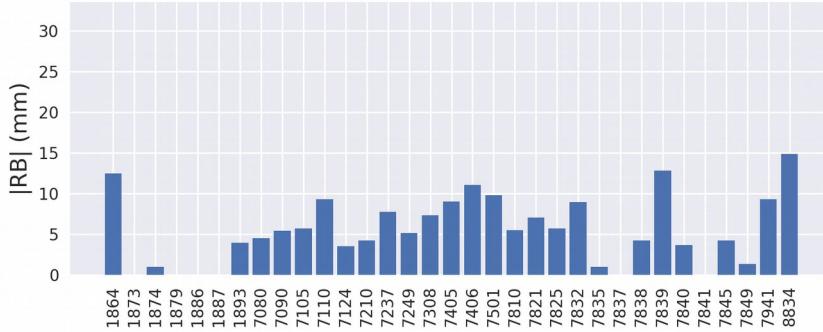
LAGEOS



LAGEOS-2



ETALON



Range biases do **NOT**  
disappear using the  
new CoM values



# Summary

We have an improved CoM modelling for the spherical geodetic satellites

Updated modelling takes into consideration more details about the measuring process

Results introduce significant differences in the CoM offsets for LAGEOS, LAGEOS-2, Etalon and AJISAI

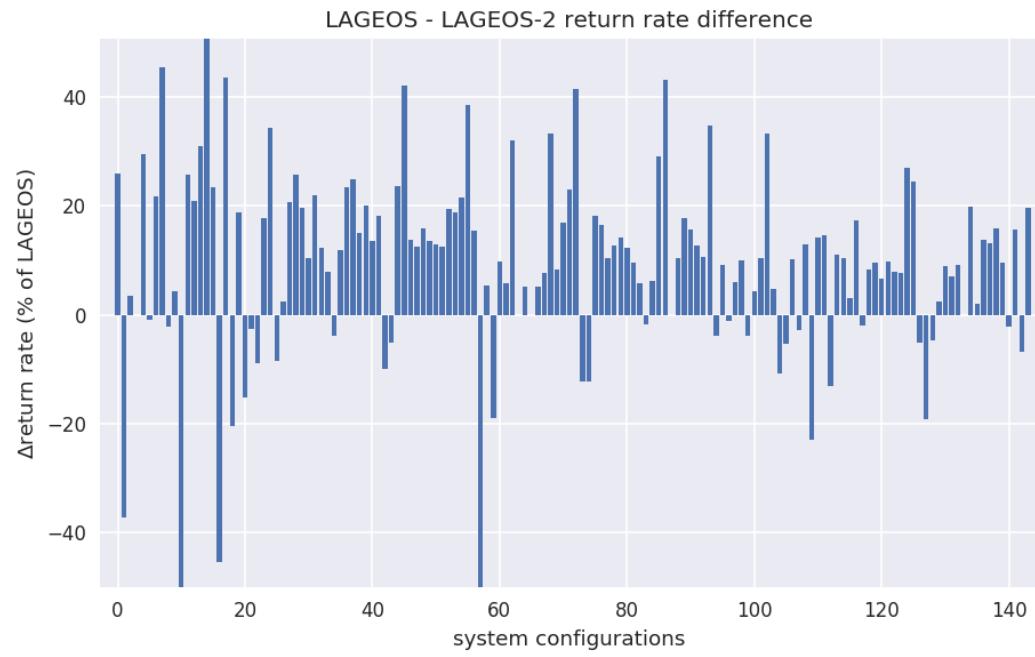
Differences for LAGEOS introduce a scale change in the SLR frame

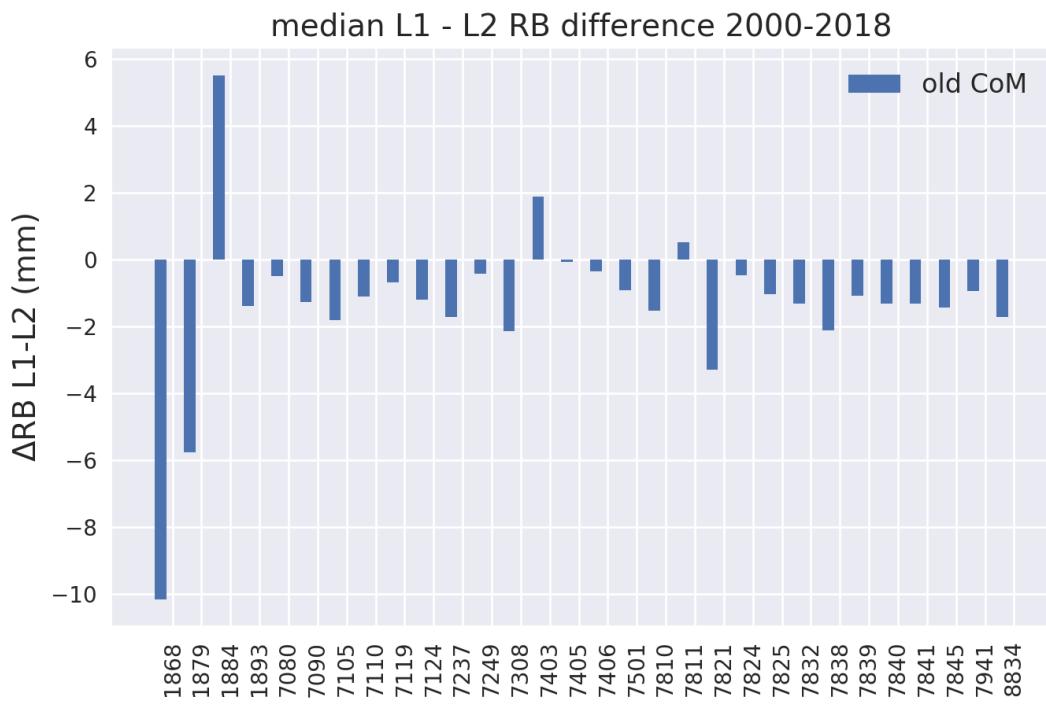
## Caveats:

- incomplete knowledge of some systems details
- model only considers ideal, linear behaviour
- realistically, accuracy no better than ~2-3 mm for LAGEOS and ~6 mm for Etalon/AJISAI on average across the network; much worse for individual cases

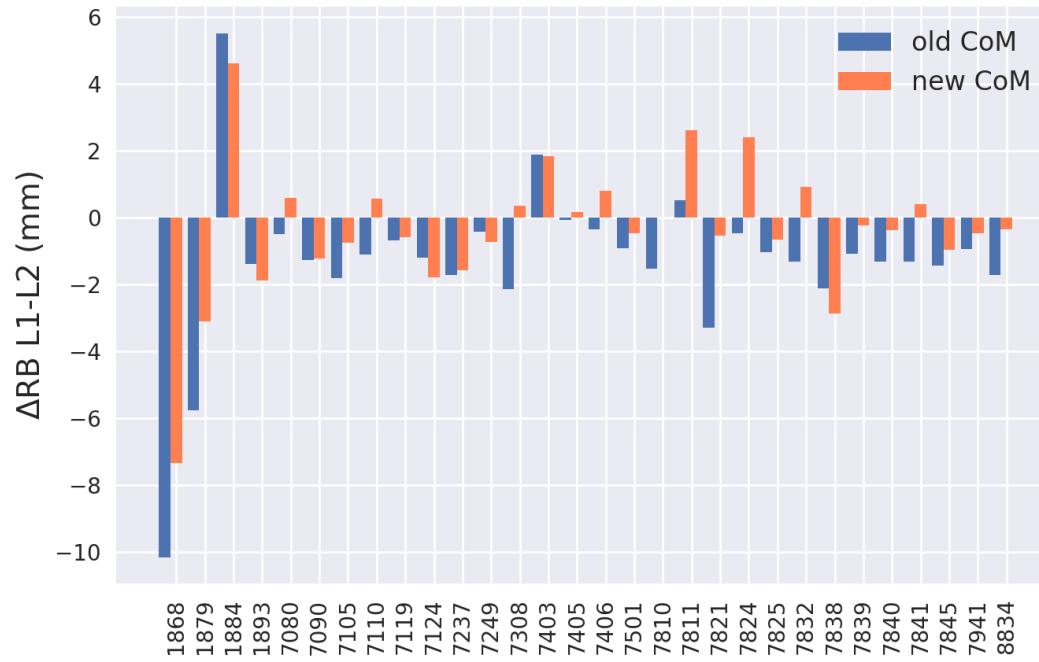
CoM alone **can not** possibly fix everything we see in the orbital solutions

# Thank you





median L1 - L2 RB difference 2000-2018



### 3. CoM computation (multi-photon systems)

Station	epochs	mirror diam	laser wave	eng	width	rate	detector type/model	qe	rise	jitt	timer model	prec	policy/reduc.
7941 MATM	20100524	20500101	150	532	100	50	MCP PMT210	15	120	30	ET HTSI	2	cal sat

