AstroWidget. Position Calculations 1

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Contents

1 Introduction

Astrowidget is a package created to aid in the astronomical observations of celestial objects. It calculates the position of sources and makes transformations of coordinates. This package is a partial result of the work done to write a control program for the 14m radiotelescope at CAY.

Astrowidget is a collection of C programs and functions mostly written from scratch following some excellent books on astronomy (see bibliography). I chose C as the programming language because the control of the 14m radiotelescope will be performed from a VME machine with a real time OS in Unix, with a C compiler. In order to keep the maximum compatibilty I have used Ansi C.

All functions have been grouped in libraries and the different programs have been glued in a single package using Tcl/Tk (see http://sunscript.sun.com) Tcl/Tk, is a very powerful script language which provides a very comfortable and nice Graphical User Interface. This feature makes Astrowigdet easy to use and to port to other Unix OSs.

This report only concentrates on the lower layer of AstroWidget. That is, the basic functions used for computing positions and making transformations of coordinates.

2 Coordinate systems

In astronomy there are 4 basic coordinate systems which may be distinguished by the origin of the coordinate system and the XY plane. In all cases except in the topocentric system, the X axis is given by the intersection of the ecliptic with the true equator.

- heliocentric equatorial. The origin is the center of the sun. The XY plane is the true equator. This is seldom used.
- geocentric equatorial. The origin is the center of the earth. The XY plane is the true equator. The cartesian coordinates are written in the following way:

$$x = r\cos\delta\cos\alpha \quad (1)$$
$$y = r\cos\delta\sin\alpha \quad (2)$$
$$z = r\sin\delta \quad (3)$$

  α and δ are usually known as rigth ascention and declination. This is the most widely system because it is used for galactic and extragalactic sources.

- heliocentric ecliptic. The origin is the center of the sun. The XY plane is the ecliptic. The cartesian coordinates are written as follows,

$$x = r\cos b\cos l \quad (4)$$
$$y = r\cos b\sin l \quad (5)$$
$$z = r\sin b \quad (6)$$

  where l and b are the ecliptic longitude and ecliptic latitude respectively. This is the usual system for planets and other objects in the solar system.

- geocentric ecliptic. The origin is the center of the earth, and the XY plane is the ecliptic. The cartesian coordinates are written as,

$$x = r\cos\beta\cos\lambda \quad (7)$$
$$y = r\cos\beta\sin\lambda \quad (8)$$
$$z = r\sin\beta \quad (9)$$

where λ and β are the ecliptic longitude and ecliptic latitude respectively. This system is also used for objects in the solar system.

- topocentric. The origin is the local place of observation on the surface of the Earth, the XY plane is the horizon and the X axis the local meridian. The cartesian coordinates are written as,

$$x = r\cos Az\cos El \quad (10)$$
$$y = r\cos Az\sin El \quad (11)$$
$$z = r\sin El \quad (12)$$

where Az and El are the azimuth and elevation of the source latitude respectively.

The true equator is affected by the nutation. We remind here that there is no *true* ecliptic, since the ecliptic is defined by the regular motion of the earth around the sun.

Since these systems are linked to the Earth they depend on time. Therefore it is the common practice to refer the coordinates of celestial objects to standard epochs, like B1950 or J2000, and then transform to the desired epoch.

In order to go from the equatorial to the ecliptic coordinate system one has to make a rotation. If one desires to go from heliocentric to geocentric it is necessary to make a translation from the baricenter of the solar system to the center of the earth. This implies to calculate the position of the earth relative to the sun. All transformations can be easily described within a vectorial space. Let (x_g, y_g, z_g) be the coordinates of an object in the equatorial geocentric system for a certain epoch. The coordinates in the ecliptic heliocentric system for the same epoch should be:

$$\begin{pmatrix} x_h \\ y_h \\ z_h \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon & \sin\varepsilon \\ 0 & -\sin\varepsilon & \cos\varepsilon \end{pmatrix} \begin{pmatrix} x_g \\ y_g \\ z_g \end{pmatrix} + \begin{pmatrix} x_\oplus \\ y_\oplus \\ z_\oplus \end{pmatrix}$$

where ε is the obliquity of the ecliptic and $(x_\oplus, y_\oplus, z_\oplus)$ are the coordinates of the Earth referred to an ecliptic heliocentric coordinate system. The obliquity of the ecliptic depends on time and does not have an exact expression. However we use the following formula, adopted by the IAU [**?**],

$$\varepsilon = \varepsilon_0 + \delta\varepsilon \quad (13)$$
$$\varepsilon_0 = 23^\circ 26' 21''.448 - 46''.8150 \cdot T - 0''.00059 T^2 + 0''.001813 T^3 \quad (14)$$

where T is the time in julian centuries from the epoch J2000, and where $\delta\varepsilon$ is obtained when the nutation is computed. This expression has an error of 1 arcsec over a period of 2000 years, and 10 arcsec over a period of 4000 years. Figure **??** shows the variation of the obliquity in an interval of 20000 years around J2000, and the approximation that we use.

3 Time

In Physics the second is the fundamental time unit. It is provided by a Cesium clock. The scale associated with it is called TAI (International Atomic Time). However in astronomy, the earth rotation and its rotation around the sun, are fundamental for the observation of celestial bodies. The scale associated with the earth rotation is called UT1 (Universal Time). The 1 suffix accounts for many corrections to the earth rotation. See [**?**] for an explanation. There is a third scale which is the one used for legal purposes, called UTC, which links UT1 and TAI. At any time the UTC differs a integer number of seconds from

TAI and less than a second from UT1. Therefore the UTC jumps 1 second each 1 or 2 years. When we compute an astronomical event we will use the UTC scale, since it is the legal one. That means that we will have to supply DUT1=UT1−UTC.

The origin of the time scale as accorded by the IAU is at 12 AM of January 1, on year −4712 (juliano calendar), at the Greenwich meridian. Days counted from this origin are called Julian days (JD). One days has 86400 seconds. One of the fundamental epochs is J2000, which corresponds to January 1, at 12 AM at the Greenwich meridian on year 2000. Another fundamental epoch is B1950 which corresponds to Julian Day 2433282.423. Sometimes a Modified Julian Day is used. The advantage is that the number is smaller.

$$\mathrm{MJD} = \mathrm{JD} - 2400000.5 \tag{15}$$

Through all the calculations we will use an algorithm taken from [?] to convert from the civil calendar to Julian day and back. We have used day October 12, 1582, as the time when the gregorian calendar began to be used. This date differs from that used by the english colonies which adhered to the calendar much later.

3.1 Civil Calendar to Julian Day

(16)

3.2 Julian Day to Civil Calendar

(17)

4 Precession

The gravitational pull of the Sun and the Moon on a non spherical body produces a torque which causes a movement of the body's rotation axis. The Earth is an oblate body and the movement of its axis can be splited artificially in a long-term periodic (secular) movement, and a short-term periodic one. The first one is called luni-solar precession and makes the axis of the Earth rotate slowly around another axis. This effect causes the ecliptic longitude of a celestial object to change, while its ecliptic latitude stays fixed. The gravitational influence of the planets also affects the movement of the Earth. They cause the earth to slightly depart from the elliptic orbit it had if there were no planets, and therefore the pole of the ecliptic changes slightly. This effect causes the equatorial declination of a celestial object to be fixed while the right ascention changes with time. Since these two terms are long term they have been grouped in a single expression called general precession. The general precession formula describes the change of the Earth axis from an initial to a final epoch given by Julian Days JD_i and JD_f respectively. We give here the general precession expression in cartesian equatorial coordinates.

Let T be,

$$T = \frac{\mathrm{JD}_i - \mathrm{JD2000}}{36525} \quad / \quad \mathrm{JD2000} = 2451545.0 \tag{18}$$

$$t = \frac{\mathrm{JD}_f - \mathrm{JD}_i}{36525} \tag{19}$$

Let the variables χ, z and θ be

$$\begin{aligned}
\chi &= (2306.2181 + 1.39656T - 0.000139T^2)t + (0.30188 - 0.000344T)t^2 + 0.017998t^3 \\
z &= (2306.2181 + 1.39656T - 0.000139T^2)t + (1.09468 + 0.000066T)t^2 + 0.018203t^3 \\
\theta &= (2004.3109 - 0.85330T - 0.000217T^2)t - (0.42665 + 0.000217T)t^2 - 0.041833t^3
\end{aligned}$$

where the units are sexagesimal arcseconds

Let x_i, y_i and z_i be the initial cartesian equatorial coordinates of the object and x_f, y_f and z_f the final cartesian equatorial coordinates. Then

$$\begin{pmatrix} x_f \\ y_f \\ z_f \end{pmatrix} = \begin{pmatrix} -\sin\chi\sin z + \cos\chi\cos z\cos\theta & -\cos\chi\sin z - \sin\chi\cos z\cos\theta & -\cos z\sin\theta \\ \sin\chi\cos z + \cos\chi\sin z\cos\theta & \cos\chi\cos z - \sin\chi\sin z\cos\theta & -\sin z\sin\theta \\ \cos\chi\sin\theta & -\sin\chi\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}$$

The previous expressions should only be used to objects referred to the IAU system at J2000, called the FK5 system. The previous system called FK4 (B1950), differs from the FK5 one, in two items.

1. The origin for right ascentions of the FK4 has a small error (estimar).
2. The E terms due to the anual aberration were included in the mean positions of objects because they are almost constant with time. Now they are not.

We have not included a way to correct from the old FK4 catalog to the new one. The order of magnitude of the correction is xxxxx.

The precission of the algorithms that we use is yyyy as compared with those from USNO (US Naval Observatory).

The order of magnitude of precession is 50 arcsecs per year. The intersection of the equator with the ecliptic regresses about 50 arcsec per year along the ecliptic, while the ecliptic tilts *presently* at a speed of 0.47 arcseconds per year. This movement is not linear but periodic.

5 Nutation

The nutation is the periodic short-term movement of the earth's axis that we mentioned in the previous section. The instantaneous axis of rotation of the Earth oscillates around a mean axis. This movement can be described by a sum of periodic terms, of which the most important one has a period of 18.6 years and an amplitude of 20 arcseconds approximately. The nutation can be splitted in two components. The nutation in longitude $\Delta\psi$ affects the (ecliptic) longitude of objects. The nutation in obliquity $\Delta\varepsilon$ affects the obliquity of the ecliptic.

The algorithm that we use is taken from the IAU [], and we have used the arguments provided by program **xephem 3.0**. Let us define the following variables:

D	Mean elongation of the Moon from the Sun
M	Mean anomaly of the Sun
M'	Mean anomaly of the Moon
F	Moon's argument of latitude
Ω	Longitude of the ascending node of the Moon's mean orbit on the ecliptic.

Argument	a	b	c	d
D	1072261.307	1602961601.328	-6.891	0.019
M	1287099.804	129596581.224	-0.577	-0.012
M'	485866.733	1717915922.633	31.310	0.064
F	335778.877	1739527263.137	-13.257	0.011
Ω	450160.280	-6962890.539	7.455	0.008

Table 1: *Arguments of the parameters in arcseconds*

All of them have the following dependence with time T,

$$f(T) = a + bT + cT^2 + dT^3 \tag{20}$$

where T is given by equation 18 Table **??** summarizes the values of parameters a, b, c and d, for the arguments previously mentioned.

The nutation is then obtained by making the sum of the terms given in table **??**. Let us take as example the second line of the table:

$$\Delta\psi_2 = (1.3187 - 1.6T)\sin(-2D + 2F + 2\Omega) \tag{21}$$

$$\Delta\varepsilon_2 = (0.5736 - 3.1T)\cos(-2D + 2F + 2\Omega) \tag{22}$$

(23)

Then the total nutation in longitude and obliquity would be,

$$\Delta\psi = \sum_i \Delta\psi_i \tag{24}$$

$$\Delta\varepsilon = \sum_i \Delta\varepsilon_i \tag{25}$$

Tables **?? ??** summarize the periodic terms for the nutation in longitude and obliquity which should be used in the sum.

The corrections to take into account the nutation are done using cartesian coordinates and the nutation matrix, Let dx, dy and dz be the differential corrections to be applied to the initial cartesian equatorial coordinates of the object (x_i,y_i,z_i),

$$\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} 0 & -\delta\psi\cos\varepsilon_0 & -\delta\psi\sin\varepsilon_0 \\ \delta\psi\cos\varepsilon_0 & 0 & -\delta\varepsilon \\ \delta\psi\sin\varepsilon_0 & \delta\varepsilon & 0 \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} \tag{26}$$

Order of magnitude of the corrections.

6 Anual Aberration

We call anual aberration the effect which causes a change in the coordinates of a source as seen from the Earth due to the movement of the Earth around the Sun and the finite value of the speed of light. The order of magnitude of the change is approximately 20 arcsecs ($\sim V_{\oplus}/c$). In order to get a rigorous exact value it is necessary to determine the true motion of the Earth referred to the Sun. The true motion is an

D	M	M'	F	Ω	ψ_0	ψ_t	ε_0	ε_t
0	0	0	0	1	-171996	-1742	92025	89
0	0	0	0	2	2062	2	-895	5
-2	0	2	0	1	46	-24		
2	0	-2	0	0	11	0		
-2	0	2	0	2	-3	1		
1	-1	0	-1	0	-3	0		
0	-2	2	-2	1	-2	1		
2	0	-2	0	1	1	0		
0	0	2	-2	2	-13187	-16	5736	-31
0	1	0	0	0	1426	-34	54	-1
0	1	2	-2	2	-517	12	224	-6
0	-1	2	-2	2	217	-5	-95	3
0	0	2	-2	1	129	1	-70	0
2	0	0	-2	0	48	1		
0	0	2	-2	0	-22	0		
0	2	0	0	0	17	-1	0	0
0	1	0	0	1	-15	9		
0	2	2	-2	2	-16	1	7	0
0	-1	0	0	1	-12	6		
-2	0	0	2	1	-6	3		
0	-1	2	-2	1	-5	3		
2	0	0	-2	1	4	-2		
0	1	2	-2	1	4	-2		
1	0	0	-1	0	-4	0		
2	1	0	-2	0	1	0		
0	0	-2	2	1	1	0		
0	1	-2	2	0	-1	0		
0	1	0	0	2	1	0		
-1	0	0	1	1	1	0		
0	1	2	-2	0	-1	0		
0	0	2	0	2	-2274	-2	977	-5
1	0	0	0	0	712	1	-7	0
0	0	2	0	1	-386	-4	200	0
1	0	2	0	2	-301	0	129	-1
1	0	0	-2	0	-158	-1		
-1	0	2	0	2	123	-53		
0	0	0	2	0	63	-2		
1	0	0	0	1	63	1	-33	0
-1	0	0	0	1	-58	-1	32	0
-1	0	2	2	2	-59	26		
1	0	2	0	1	-51	27		
0	0	2	2	2	-38	16		
2	0	0	0	0	29	-1		
1	0	2	-2	2	29	-12		

Table 2: *Argument parameters in arcseconds (I)*

D	M	M'	F	Ω	ψ_0	ψ_t	ε_0	ε_t
2	0	2	0	2	-31	13		
0	0	2	0	0	26	-1		
-1	0	2	0	1	21	-10		
-1	0	0	2	1	16	-8		
1	0	0	-2	1	-13	7		
-1	0	2	2	1	-10	5		
1	1	0	-2	0	-7	0		
0	1	2	0	2	7	-3		
0	-1	2	0	2	-7	3		
1	0	2	2	2	-8	3		
1	0	0	2	0	6	0		
2	0	2	-2	2	6	-3		
0	0	0	2	1	-6	3		
0	0	2	2	1	-7	3		
1	0	2	-2	1	6	-3		
0	0	0	-2	1	-5	3		
1	-1	0	0	0	5	0		
2	0	2	0	1	-5	3		
0	1	0	-2	0	-4	0		
1	0	-2	0	0	4	0		
0	0	0	1	0	-4	0		
1	1	0	0	0	-3	0		
1	0	2	0	0	3	0		
1	-1	2	0	2	-3	1		
-1	-1	2	2	2	-3	1		
-2	0	0	0	1	-2	1		
3	0	2	0	2	-3	1		
0	0	0	0	1	-171996	-1742	92025	89
0	-1	2	2	2	-3	1		
1	1	2	0	2	2	-1		
-1	0	2	-2	1	-2	1		
2	0	0	0	1	2	-1		
1	0	0	0	2	-2	1		
3	0	0	0	0	2	0		
0	0	2	1	2	2	-1		
-1	0	0	0	2	1	-1		
1	0	0	-4	0	-1	0		
-2	0	2	2	2	1	-1		
-1	0	2	4	2	-2	1		
2	0	0	-4	0	-1	0		
1	1	2	-2	2	1	-1		
1	0	2	2	1	-1	1		
-2	0	2	4	2	-1	1		
-1	0	4	0	2	1	0		

Table 3: *Argument Parameters in arcseconds (II)*

D	M	M'	F	Ω	ψ_0	ψ_t	ε_0	ε_t
2	0	2	0	2	-31	13		
1	-1	0	-2	0	1	0		
2	0	2	-2	1	1	-1		
2	0	2	2	2	-1	0		
1	0	0	2	1	-1	0		
0	0	4	-2	2	1	0		
3	0	2	-2	2	1	0		
1	0	2	-2	0	-1	0		
0	1	2	0	1	1	0		
-1	-1	0	2	1	1	0		
0	0	-2	0	1	-1	0		
0	0	2	-1	2	-1	0		
0	1	0	2	0	-1	0		
1	0	-2	-2	0	-1	0		
0	-1	2	0	1	-1	0		
1	1	0	-2	1	-1	0		
1	0	-2	2	0	-1	0		
2	0	0	2	0	1	0		
0	0	2	4	2	-1	0		
0	1	0	1	0	1	0		

Table 4: *Argument Parameters in arcseconds (II)*

elliptical orbit perturbed by the gravitational pull of the Moon and the planets. One also has to take into account that the Sun moves around the solar system barycenter due mainly to the action of Jupiter and Saturn.

Ron and Vondrák [**?**] have presented an algorithm to compute the Earth movement. Let us define the mean longitude of the planets and the moon in radians as shown in table **??**, where T is defined by equation 18,

The arguments in table **??** are referred to the J2000 epoch. In order to compute the cartesian components of the earth velocity one has to make the following sum,

$$V_\oplus = \begin{pmatrix} \sum_i a_x \cos A_i + b_x \sin A_i \\ \sum_i a_y \cos A_i + b_y \sin A_i \\ \sum_i a_z \cos A_i + b_z \sin A_i \end{pmatrix}$$

where all variables are defined in table **??**

The correction to be applied to the J2000 right ascention and declination of an object is,

$$\Delta\alpha = \frac{v_y \cos\alpha - v_x \sin\alpha}{c\cos\delta} \quad (27)$$

$$\Delta\delta = \frac{(v_x \cos\alpha + v_y \sin\alpha)\sin\delta - v_z \cos\delta}{c} \quad (28)$$

Planet	Argument			
Venus	L_2	3.1761467	+	1021.3285546 T
Mars	L_3	1.7534703	+	628.3075849 T
Sun	L_4	6.2034809	+	334.0612431 T
Jupiter	L_5	0.5995465	+	52.9690965 T
Saturn	L_6	0.8740168	+	21.3299095 T
Uranus	L_7	5.4812939	+	7.4781599 T
Neptune	L_8	5.3118863	+	3.8133036 T
Moon	L'	3.8103444	+	8399.6847337 T
Moon	D	5.1984667	+	7771.3771486 T
Moon	M'	2.3555559	+	8328.6914289 T
Moon	F	1.6279052	+	8433.4661601 T

Table 5: *Arguments in radians*

7 Local Standard of Rest

The Local Standard of Rest (LSR) is the centroid of motion of the local group of stars in the neighbourhood of the sun. Therefore the sun moves towards (or away from) this place. There are several coordinates for the solar apex (see [**?**]). We will use the radioastronomical position,

$\alpha_\odot$ = 18h 3′ 50.2″ (J2000)
$\delta_\odot$ = 30° 0′ 16.8″ (J2000)
$V_\odot$ = 20 km/s

Since the speed is positive the sun is moving away from the LSR.

Radioastronomical observations need the radial velocity of the observed object to take into account the Doppler effect. It is usual to refer the radial velocity of a celestial object to the LSR. That means that we have to refer the radial velocity of an object first to the center of the sun (heliocentric velocity), after to the center of erath (geocentric velocity) and finally to the topocentric place where the observation will take place. Conversions are easely computed if we know the movement of the Sun and the Earth and the place we will observe from. It is justed done by making scalar products.

$$V_{lsr} = V_{hel} + \mathbf{V}_{sun} \cdot \mathbf{u}_{radial} \tag{29}$$

where V_{lsr} and V_{hel} are the radial velocities of the source referred to the LSR and to the center of sun respectively. However the origin of the coordinate system in both cases is the center of the earth and therefore *radial* means as seen from the center of the erath. $\mathbf{V}_{sun}$ is a vector which points towards the LSR and whose magnitude is 20 km/s. $\mathbf{u}_r adial$ is a unitary vector in the source direction. If we use equatorial geocentric coordinates the helicentric velocity is,

$$V_{hel} = V_{lsr} - V_\odot(\cos\delta_\odot\cos\alpha_\odot, \cos\delta_\odot\sin\alpha_\odot, \sin\delta_\odot) \cdot \begin{pmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{pmatrix} \tag{30}$$

where α and δ are the right ascention and declination of the source.

To obtain the geocentric radial velocity,

$$V_{hel} = V_{geo} + \mathbf{V}_\oplus \cdot \mathbf{u}_{radial} \tag{31}$$

i	A	a_x	b_x	a_y	b_y	a_z	b_z
1	L_3	-1719914-2T	-25	25-13T	1578089+156T	10+32T	684185-358T
2	$2L_3$	6434+141T	28007-107T	25697-95T	-5904-130T	11141-48T	-2559-55T
3	L_5	715	0	6	-657	-15	-282
4	L'	715	0	0	-656	0	-285
5	$3L_3$	486-5T	-236-4T	-216-4T	-446+5T	-94	-193
6	L_6	159	0	2	-147	-6	-61
7	F	0	0	0	26	0	-59
8	$L' + M'$	39	0	0	-36	0	-16
9	$2L_5$	33	-10	-9	-30	-5	-13
10	$2L_3 - L_5$	31	1	1	-28	0	-12
11	$3L_3 - 8L_4 + 3L_5$	8	-28	25	8	11	3
12	$5L_3 - 8L_4 + 3L_5$	8	-28	-25	-8	-11	-3
13	$2L_2 - L_3$	21	0	0	-19	0	-8
14	L_2	-19	0	0	17	0	8
15	L_7	17	0	0	-16	0	-7
16	$L_3 - 2L_5$	16	0	0	15	1	7
17	L_8	16	0	1	-15	-3	-6
18	$L_3 + L_5$	11	-1	-1	-10	-1	-5
19	$2L_2 - 2L_3$	0	-11	-10	0	-4	0
20	$L_3 - L_5$	-11	-2	-2	9	-1	4
21	$4L_3$	-7	-8	-8	6	-3	3
22	$3L_3 - 2L_5$	-10	0	0	9	0	4
23	$L_2 - 2L_3$	-9	0	0	-9	0	-4
24	$2L_2 - 3L_3$	-9	0	0	-8	0	-4
25	$2L_6$	0	-9	-8	0	-3	0
26	$2L_2 - 4L_3$	0	-9	8	0	3	0
27	$3L_3 - 2L_4$	8	0	0	-8	0	-3
28	$L' + 2D - M'$	8	0	0	-7	0	-3
29	$8L_2 - 12L_3$	-4	-7	-6	4	-3	2
30	$8L_2 - 14L_3$	-4	-7	6	-4	3	-2
31	$2L_4$	-6	-5	-4	5	-2	2
32	$3L_2 - 4L_3$	-1	-1	-2	-7	1	-4
33	$2L_3 - 2L_5$	4	-6	-5	-4	-2	-2
34	$3L_2 - 3L_3$	0	-7	-6	0	-3	0
35	$2L_3 - 2L_4$	5	-5	-4	-5	-2	-2
36	$L' - 2D$	5	0	0	-5	0	-2

Table 6: *Velocity components of the earth referred to the barycenter of the solar system and for the J2000 epoch. Units are* 10^{-8} *AU/day*

where V_{geo} is the velocity of the source referred to the center of the earth and $\mathbf{V}_{\oplus}$ the velocity of the earth. The geocentric velocity is then,

$$v_{geo} = V_{hel} - (v_x, v_y, v_z) \cdot \begin{pmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{pmatrix} \tag{32}$$

where v_x, v_y and v_z are the cartesian components of the velocity of the earth in a equatorial geocentric coordiante system at epoch J2000. The velocity of the earth is computed using the Ron-Vondrák algorithm, which we described in the previous section.

To obtain the topocentric velocity of a source,

$$V_{geo} = V_{topo} + \mathbf{V}_{local} \cdot \mathbf{u}_{radial} \tag{33}$$

where V_{topo} is the velocity of the source referred to the local place of observation and $\mathbf{V}_{local}$ the velocity of the local place. The topocentric velocity is then,

$$v_{topo} = V_{geo} - (v_{lx}, v_{ly}, v_{lz}) \cdot \begin{pmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{pmatrix} \tag{34}$$

where,

$$v_{lx} = \frac{dx_l}{dt} = -D_0 w_{\oplus} \sin(w_{\oplus} t) \tag{35}$$

$$v_{ly} = \frac{dy_l}{dt} = D_0 w_{\oplus} \cos(w_{\oplus} t) \tag{36}$$

$$v_{lz} = \frac{dz_l}{dt} = 0 \tag{37}$$

t is the true local sidereal time, $w_o plus$ the angular velocity of the earth x_l, y_l, z_l the cartesian coordinates the local place and D_0,

$$D_0 = \sqrt{x_l^2 + y_l^2 + z_l^2} \tag{38}$$

The order of magnitude of the previous transformations is offered in table xxxx

8 Parallax

Parallax is an effect that affects the coordinates of objects due to the displacement of the earth along its orbit. Stellar parallaxes are never higher than $0.8''$. One should take into account the parallax if a great accuracy is needed and even then only a low number of stars are affected. According to the Hipparcos catalog.

The way to calculate parallax is simple. One needs to know the position of the Earth for the epoch of the observation ($R_o^{obs} plus$) and for the epoch to which the position of the source is referred to ($R_o^{ref} plus$). And then calculate the difference.

$$R_{\oplus}^{obs} = R_{\oplus}^{ref} + D(\cos\delta\cos\alpha, \cos\delta\sin\alpha, \sin\delta) \tag{39}$$

where D is the distance of the source to the earth at the J2000 epoch, and it is computed, from the ecliptic latitude (β), the parallax (which is usually given in milliarcseconds) and the major axis of the earth orbit,

$$D = \frac{S \cos \beta}{\mathrm{parallax}} \tag{40}$$

The position of the Earth for the two different epochs is obtained using either the Chapront solution or the VSOP87 algorithm. See section on planets for an explanation.

9 Sidereal time

The sidereal time is a different time scale used in astronomy. This scale relates the rotation of the Earth with the stars, and not with the Sun as the UT1 does. Therefore each complete revolution of the Earth takes less than 24 hours to complete. The exact definition of the local sidereal time is given by,

$$\theta = \mathrm{AH} + \alpha \tag{41}$$

where AH is the hour angle (measured from the instersection of the true equator with the ecliptic) of a source and α is its right ascention. If one considers the medium equator and ecliptic the mean sidereal time at Greenwich (θ_0) is given by the following expression (IAU 1982), which relates the UT1 scale with the sidereal time one,

$$\theta_0 = 6^h + 41^m + 50.54841 + 8640184.812866T + 0.093104T^2 - 0.0000062T^3 \tag{42}$$

where T is given by equation 18 and the units of the previous expression are seconds. In order to get the apparent Greenwich sidereal time,

$$\theta_g = \theta_0 + \frac{\Delta\psi \cos \epsilon}{15} \tag{43}$$

where $\Delta\psi$ is the nutation in longitude (given in arcseconds) and ϵ the true obliquity. To obtain the local sidereal time, at a place with longitude λ,

$$\theta = \theta_g + \lambda \tag{44}$$

where λ is positive towards the east of the Greenwich meridian

10 Apparent Coordinates

The apparent coordinates of a celestial object at a certain time are its geocentric equatorial coordinates referred to the true equator and ecliptic. The corrections that have to be applied are, in the correct order:

1. FK5 coordinates of the object
2. The proper motion of the source
3. The effect of anual aberration as explained in section xxx using the Ron-Vondrak algorithm
4. The precession
5. The nutation
6. the anual parallax

The algortihms to be used have already been described in the previous sections.

11 Topocentric Coordinates

The topocentric coordinates of an object are local and therefore referred to the place where astronomical observations are performed. The transformation from equatorial geocentric coordinates to topocentric one involves a translation from the center of the Earth to the local place and a rotation of the coordinate system to take into account that azimuth is referred to the local meridian and elevation to the local horizon. Let x_p, y_p and z_p be the cartesian coordinates of the local place, and θ be the local sidereal time and α the right ascention of the source, then the hour angle (AH) is given by,

$$\mathrm{HA} = \theta - \alpha \tag{45}$$

and the cartesian coordinates of the source in the local system is,

$$\begin{pmatrix} x_l \\ y_l \\ z_l \end{pmatrix} = \begin{pmatrix} \cos\phi\cos AH & -\cos\phi\sin AH & \sin\phi\cos AH \\ \cos\phi\sin AH & -\cos\phi\cos AH & \sin\phi\sin AH \\ -\cos\phi\sin AH & \sin\phi\sin AH & \cos\phi \end{pmatrix} \begin{pmatrix} x_g \\ y_g \\ z_g \end{pmatrix} + \begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix}$$

In order to get the topocentric velocity of the source one has to take into account the rotation of the Earth.

12 Planets

13 Equatorial to Galactic Coordinate Conversion

This is just achivied by a rotation. The North Pole of the Galactic Coordinate system is at,

$\alpha_I I$ = 17h 24′ 0.0″(B1950)
$\delta_I I$ = 00° 49′ 00″ (B1950)

and to produce a transformation,

A