Frequency switching observing mode.
Two phases calibration

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Revision history

Version	Date	Author	Updates
1.0	03-11-2016	A. Díaz-Pulido	First version
2.0	16-12-2016	A. Díaz-Pulido	Second version

Contents

1 Introduction

The resulting baseline of a frequency switching scan exhibits a lack of flatness through the whole bandwidth. This is a consecuence of the different spectral bandpass shape at the two observing frequencies and it hinds searching for new lines or the identification of weak known ones.

To overcome this problem we propose and discuss a method that calibrates both frequencies of interest.

2 Calibration procedure

When an integration is performed, the resulting spectrum is shown in arbitrary units (usually number of counts or volts). To obtain the spectra in ${T_a}^*$ or T_a (Kelvins), a calibration scan is performed before the measurement. The calibration factor is obtained from a hot and a cold measurement:

$$cal_{factor} = \frac{V_{hot} - V_{cold}}{T_{hot} - T_{cold}} \tag{1}$$

This result can be treated as an scalar or as an array. In our case cal_{factor} is an array, and provides the calibration to be applied per channel.

The system temperature can be calculated as:

$$T_{sys} = \frac{V_{cold}}{cal_{factor}} \tag{2}$$

where $cold$ could be the reference spectrum, taken near, but out of, the source of interest, and looking at the sky; hot is a spectrum looking at an absorptive material at a known temperature. Using an atmosphere model as ATM [1] we can estimate how the sky emits at a certain frequency, taking into account parameters as the atmosphere pressure, the ambient temperature and the relative humidity.

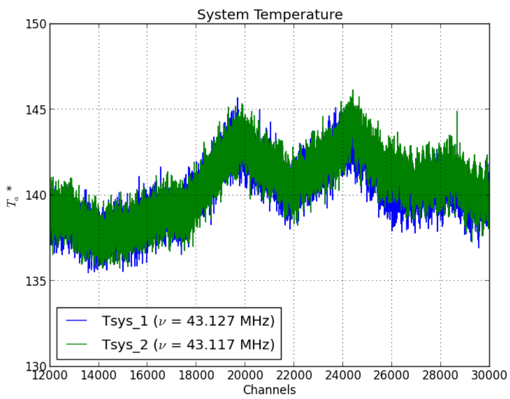

Figure 1: *System temperature for two frequencies*

Figure 1 shows the calculated system temperature array for two frequencies shifted 10 MHz. To get the calibrated source emission spectrum we use:

$$T_a^* = \frac{V_{on} - V_{cold}}{cal_{factor}} \tag{3}$$

Here, on designs the integration looking at the source of interest.

3 Frequency switching

Classically, a frequency switching scan consists in observing the source at two close frequencies and calibrating by an intermediate frequency calibration factor (or taking this factor at one of the two frequencies). This lead us to:

$$T_a(FSW) = \frac{V_1 - V_{cold}}{cal_{factor}} - \frac{V_2 - V_{cold}}{cal_{factor}} = \frac{V_1 - V_2}{cal_{factor}} \tag{4}$$

Subindex 1 stands for the upper frequency, and 2 for the lower one. The reference spectrum ($cold$) cancels out.

The main drawback of this operation is a non-flat baseline (see [2]) which arises from using a single bandpass for two frequencies. Figure 2 shows a frequency switching scan calibrated in a classic way.

3.1 Two phases calibration

To avoid the lack of flatness of the baseline we propose to calibrate each frequencies by its corresponding calibration factor (measuring hot and cold references). This leads to the next result:

$$T_a(FSW_2) = \frac{V_1 - V_{cold,1}}{cal_{factor,1}} - \frac{V_2 - V_{cold,2}}{cal_{factor,2}} \tag{5}$$

Figure 3 shows the result of using the above equation.

This result exhibits a flat baseline, with a rms $\sqrt{2}$ greater than the one obtained calibrating only at one frequency. This result agrees with theory, as we can see at appendix 4.

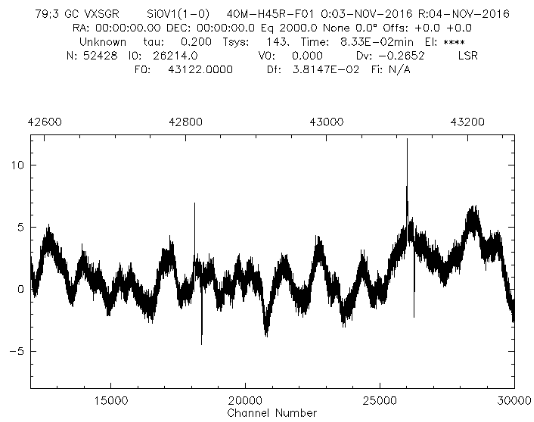

Figure 2: *Intermediate calibration factor*

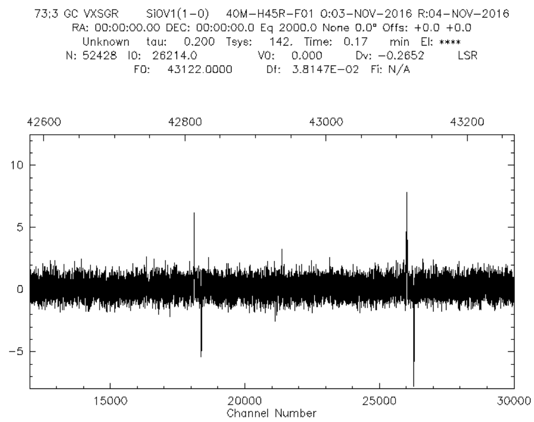

Figure 3: *Two phases calibrated*

3.2 Improving the rms

The ratio between rms_{FSW} and rms_{FSW_2} is theoretically $\frac{1}{\sqrt{2}}$ (see appendix [4]). However, we can improve the signal-to-noise ratio of a FSW_2 scan by smoothing the reference spectrum (see reference [3]). In this manner the ratio $\frac{rms_{FSW}}{rms_{FSW_2}}$ improves to get closer to 1, *i.e.*, rms_{FSW_2} decreases in a factor near $\sqrt{2}$. Figure 4 shows the experimental decrease of rms_ratio to near $\frac{1}{\sqrt{2}}$ and its later increase (as we discuss below).

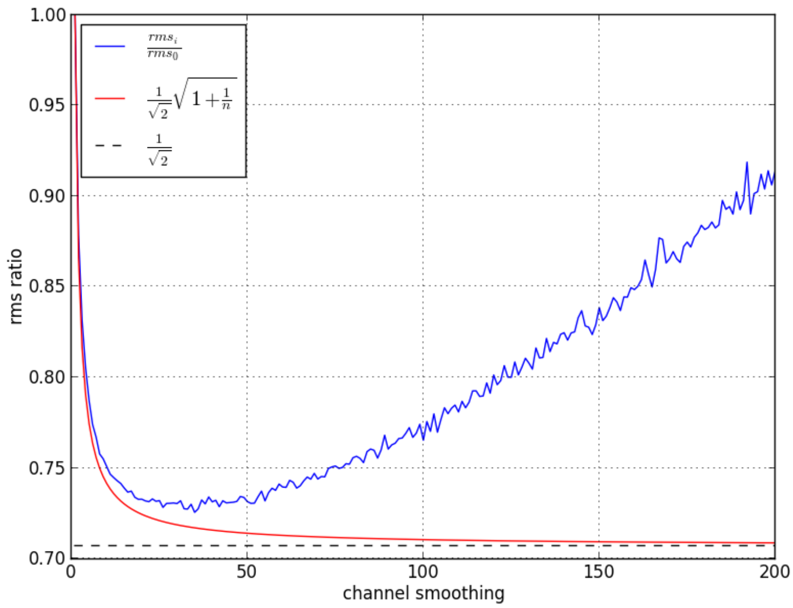

Figure 4: *Blue: rms ratio comparing a FSW using a smoothed reference respect to the same FSW without smooth the reference. Red: Theoretical reduction of rms in function of the reference channel smoothing. Dashed line: Limit to this reduction*

In theory, the expression that relates the rms of the final spectrum with the channel smoothing, tends to diminish with greater values of smoothing (Appendix 4). But here, we have to take into account why we are smoothing the reference. Substracting the reference smoothed to our integration scan, and then applying the calibration factor, leads to increase our signal-to-noise ratio (SNR) as long as both spectra holds the shape.

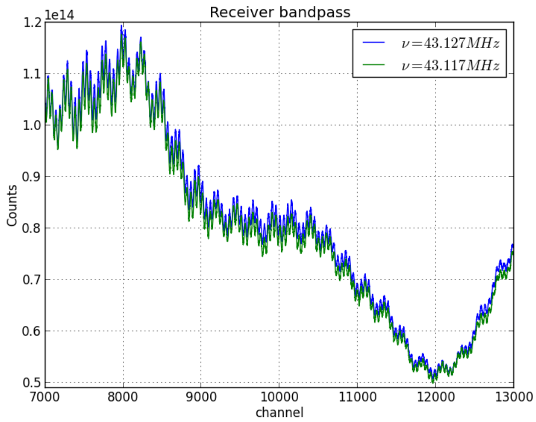

Figure 5: *spectral shape of an integration scan through a section of the band*

The spectroscopic behaviour of our band shows some ripples (as we can see at figure 5). The main component has a frequency of 1.88 MHz, or 49.33 channels for a frequency resolution of 38.15 KHz. Therefore, to sample the reference with a resolution greater to half the main component of the ripples (0.94 MHz) tends to increase the SNR of the final spectrum. This is a practical limit to smooth the reference of 25 channels.

At the figure 6 we can see an improvement of the rms in a factor 0.727 (so near to $\frac{1}{\sqrt{2}} \simeq 0.707$) smoothing the reference spectrum 25 channels.

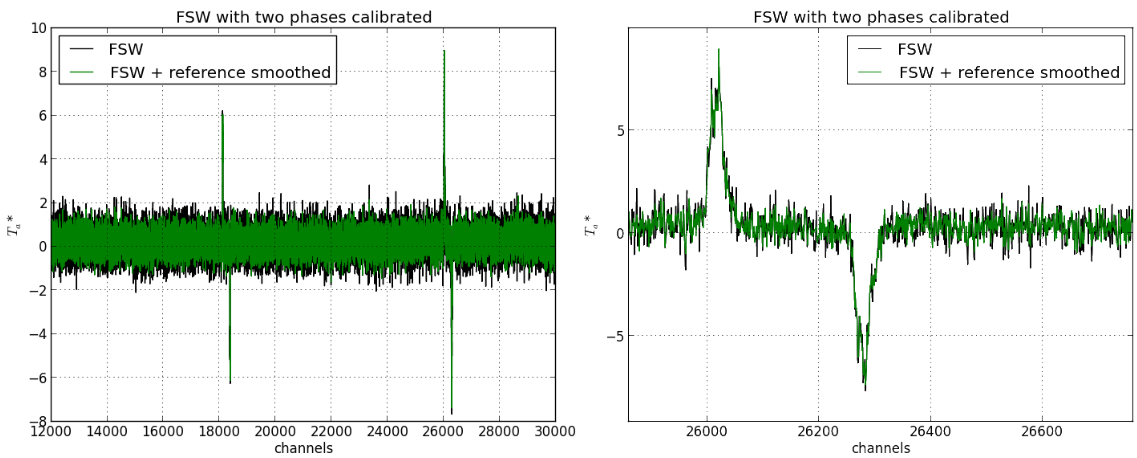

Figure 6: *Smoothing of 25 channels for the reference. Shape of the source emission do not change. Left: VX-Sgr v=1 and v=2 SiO(J=1-0) maser emission. Right: Zoom on v=1 emission.*

3.3 Conclusions

Frequency Switch scans can be improved using a separate calibration for each frequency. This policy increases the signal to noise ratio by $\sqrt{2}$ but provides flat baselines. To achieve an rms similar to the "classical" Frequency Switch one a smoothed reference spectrum can be used. This reference spectrum is used as in a position switch scan, but it requires less time. The time devoted to the reference should be $\sqrt{n}$ times less than the time integrating at the source, where n is the number of smoothed channels at the reference.

4 Appendix

When we do a position switching ($on - off$) the calibrated spectrum is obtained using:

$$T_a(on - off) = \frac{V_{on} - V_{off}}{cal_{factor}} \tag{6}$$

Where V_{on} is the power towards the source and V_{off} towards the reference. The theoretical standard deviation of the white noise (rms) is given by:

$$\sigma_{on-off} = \frac{\sqrt{2} \cdot Tsys}{\sqrt{\Delta\nu \cdot t_{on}}} \tag{7}$$

If we perform a Frequency Switching scan (eq. 4), we can achieve the same expresion. But here, we can find an advantage: if we fold the spectrum we can improve the rms in a quantity of $\frac{1}{\sqrt{2}}$. To fold a FSW scan is to make the average of the previously displaced spectra in both directions. So:

$$\sigma_{FSW_{fold}} = \frac{\sigma_{FSW}}{\sqrt{2}} = \frac{Tsys}{\sqrt{\Delta\nu \cdot t_1}} \tag{8}$$

This can be proved taking into account that the last final spectrum is a composition (an addition/substraction operation) of spectra. Knowing the rms of each single spectrum, σ_f and σ_g. The resulting rms of the final spectrum, h(f,g) is:

$$\sigma_h = \sqrt{(\frac{\partial h}{\partial f} \cdot \sigma_f)^2 + (\frac{\partial h}{\partial g} \cdot \sigma_g)^2} \tag{9}$$

In this manner, we can get the reduction of $\frac{1}{\sqrt{2}}$ when we fold a FSW scan, assuming that our final spectrum is a composition as: $h = \frac{f+g}{2}$

On the same way, when we calibrate each frequencies (eq. 5) the resulting spectrum is a composition of two spectra, $h = f - g$, each one with an rms equal to the one obtained in a FSW. So here we obtain:

$$\sigma_{FSW_2} = \sqrt{2} \cdot \sigma_{FSW} \tag{10}$$

When we fold this FSW_2 spectrum we achieve a value of rms equal to the one obtained in a position switching.

4.1 rms smoothing

Finally, smoothing the reference 3.2, the improvement in rms depends on the number of channels used.

$$T_a(FSW_2) = \frac{V_1 - V_{ref,1}}{cal_{factor,1}} - \frac{V_2 - V_{ref,2}}{cal_{factor,2}} \equiv T_a(fsw1) - T_a(fsw2) \tag{11}$$

A Frequency Switching calibrating at each frequency, without folding the result, have a rms of: $\sigma_{FSW_2} = \sqrt{2} \cdot \sigma_{fsw1} = \sqrt{2} \cdot \sigma_{fsw2} = \frac{2 \cdot Tsys}{\sqrt{\Delta\nu \cdot t_{on}}}$. Each of the terms on the subtraction are a composition of two different spectra:

$$T_a(fsw1) = \frac{V_1}{cal_{factor,1}} - \frac{V_{ref,1}}{cal_{factor,1}} \equiv v_1 - v_{ref,1} \tag{12}$$

Each one with a rms: $\sigma_{v_1} = \sigma_{v_{ref,1}} = \frac{Tsys}{\sqrt{\Delta\nu \cdot t_{on}}}$, if we integrate the same time at both phases. When we smooth a spectrum the frequency resolution takes the value $\Delta\nu \cdot n$. With that:

$$\sigma_{FSW_2} = \sqrt{2} \cdot \sigma_{fsw1} = \sqrt{2} \cdot \sqrt{\sigma_{v_1}^2 + \sigma_{v_{ref,1}}^2} = \sqrt{2} \cdot \sqrt{\frac{Tsys^2}{\Delta\nu \cdot t_{on}} + \frac{Tsys^2}{\Delta\nu \cdot n \cdot t_{on}}} \tag{13}$$

And the final theoretical rms for a FSW_2 smoothed scan is:

$$\sigma_{FSW_2}(smooth = n) = \frac{\sqrt{2} \cdot Tsys}{\sqrt{\Delta\nu \cdot t_{on}}} \cdot \sqrt{(1 + \frac{1}{n})} \tag{14}$$

References

[1] JUAN R. PARDO ET AL., *Atmospheric transmission at microwaves (ATM): An improved model for millimeter/submillimeter applications*, 2001

[2] JEFF MANGUM, NRAO, *Observing Modes Used in Radio Astronomy*, 2006

[3] JIM BRAAT, *Calibration of GBT Spectral Line Data in GBTIDL v2.1*, 2009