Thermal Conductivity and Electrical Loss of Thin Wall Millimeter Wave Stainless Steel Waveguides

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Change Record

Revision	Date	Affected Paragraphs(s)	Reason/Initiation/Remarks
A	2015-08-18	All	First Issue
B	2015-10-04	Electrical attenuation	Added measurement of attenuation in SS 304 WR10 waveguide (figure 6)

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Introduction

Thin wall stainless steel (SS) waveguides are used in cryogenic low noise receivers because they show relatively low electrical loss, almost constant with temperature, while providing some thermal isolation due to the low thermal conductivity of the material. Other possibility to obtain thermal isolation in waveguide structures is using a narrow gap between flanges with some kind of structure (choke or electromagnetic bandgap [5]) to prevent excessive radiation loss. However, these structures are more difficult to build and may still show undesired resonances in the waveguide pass band. On the other hand, stainless steel thin wall waveguides provide a perfect shield and present smooth loss and reflection. The thermal isolation of a section of SS waveguide can be raised arbitrarily by increasing its length but this also increments the electrical loss. The length of the SS waveguides in a cryogenic system should be optimized as a trade off between the thermal isolation needed and the electrical loss allowed. While the transmission loss does not change drastically with temperature, the calculation of the thermal conductivity is more complex since it depends strongly on the physical temperatures at the boundaries. This report presents the practical information needed to make accurate estimation of these parameters for commercial SS waveguides for Ka (WR-28), Q (WR-22) and W (WR-10) bands, which are often used in Yebes cryogenic receivers.

Cryogenic Thermal Conductivity

Thermal conductivity is a specific property of materials which can be defined as the heat energy transferred per unit of time and per unit of surface area divided by the temperature gradient. In SI units it is expressed in ***watts per kelvin per meter***. In the limit of infinitesimal thickness and temperature difference, the fundamental (Fourier) law of heat conduction is:

$$\dot{Q} = -k \cdot A \cdot \frac{\delta T}{\delta L} \tag{1}$$

Where:

$\dot{Q}$ = rate of heat conduction (W)
k = thermal conductivity ($W \cdot K^{-1} \cdot m^{-1}$)
A = cross-section area of the heat conduction path normal to the direction of heat flow (m^2)
T = temperature (K)
L = length (m)

Usually in simple ambient temperature calculations, the thermal conductivity is considered as a material dependant constant, but in a wide temperature ranges it should be considered a function of temperature (*k = k(T)*) [1], [2], [4]. This is particularly important at low cryogenic temperatures. If the cross section is constant then:

$$\dot{Q} = -\frac{A}{L} \cdot \int_{T_1}^{T_2} k(T)dT \tag{2}$$

Where T_1 and T_2 are the temperatures at any two points along the path of the heat flow separated by the distance L. The **thermal conductivity integral** is defined as:

$$\Theta(T_1, T_2) = \int_{T_1}^{T_2} k(T)dT \tag{3}$$

The units in the SI are ***watts per meter***. Tables or graphs of the integral as a function of temperature can be found in the literature. In these tables usually T_1 is assumed to be 0K. The value of the integral for any temperature range can be easily calculated from the tables as:

$$\int_{T_1}^{T_2} k(T)dT = \int_{T_0}^{T_2} k(T)dT - \int_{T_0}^{T_1} k(T)dT \qquad (4)$$

Thin wall stainless steel waveguides are usually manufactured with alloy 304. The wall thickness for Ka (WR-28), Q (WR-22) and W (WR-10) bands is usually 0.010” (0.254 mm). The composition of the 304 alloy is consistent with the analysis of the material of commercial SS waveguides performed by X-ray fluorescence in our lab. The thermal conductivity of stainless steel 304 has been measured and very reliable data can be found in several sources. For example, for the 1-300 K range, NIST[1] provides a 8 degree polynomial (appendix I) with a claimed error of less than 2% respect to available data (see Figure 1).

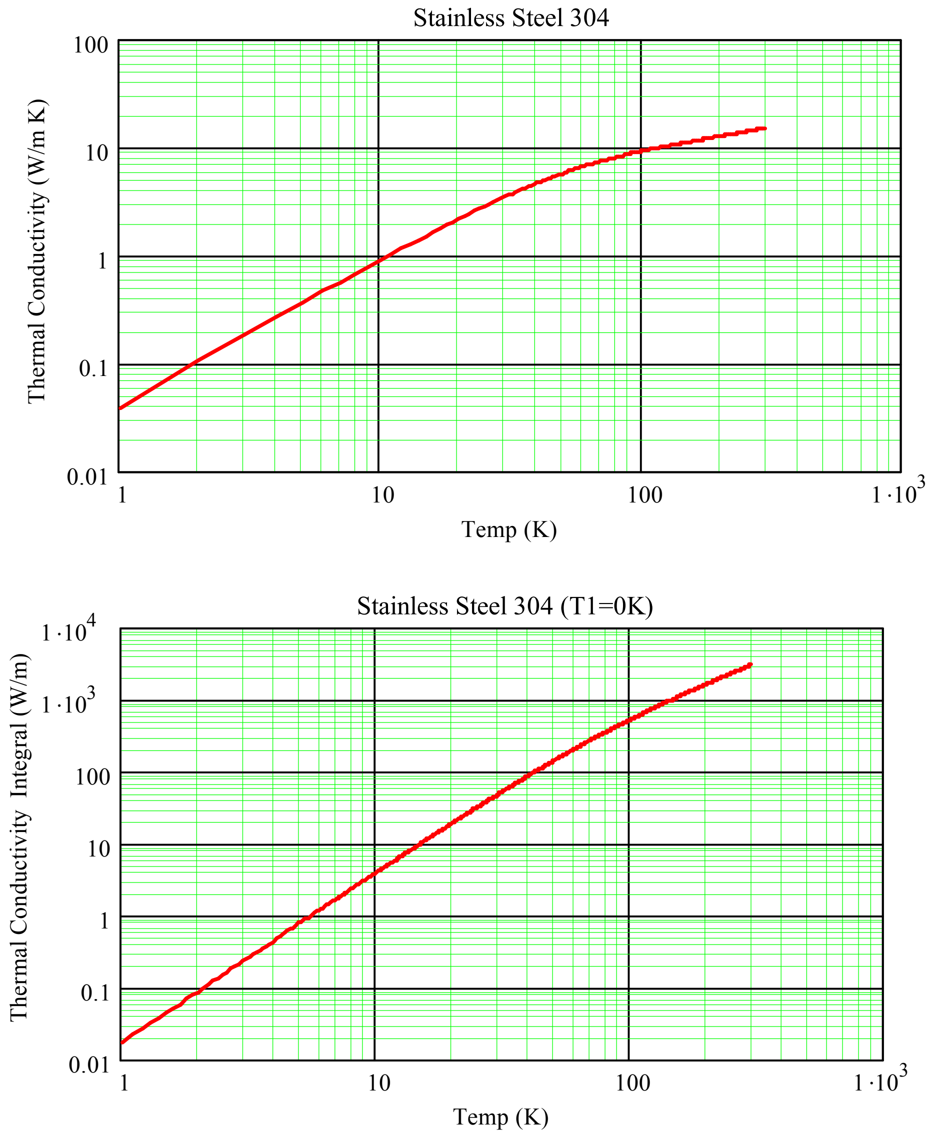

Figure 1: Thermal Conductivity and Thermal Conductivity Integral for stainless steel 304 in the 1-300 K range obtained from NIST 8 degree polynomial.

[1] NIST data from http://www.cryogenics.nist.gov/MPropsMAY/304Stainless/304Stainless_rev.htm

The thermal conductivity integral was calculated by numerical integration of the NIST polynomial in a MathCAD worksheet. Some experimental measurements of a piece of waveguide were taken in a Sumitomo RDK 415 D cryogenic cooler in order to verify the accuracy of the calculations in the case of thin wall stainless steel waveguides. The test sample consisted in a 23 mm long WR22 (Q band) waveguide soldered to two copper blocks. The bottom block was attached to the base plate of the refrigerator and the upper block was fitted with a 100 Ω resistor and a calibrated temperature sensor. The whole system was enclosed into an aluminum shield (attached to the base plate) further protected with layers of super-insulator (Al metalized Mylar) to prevent heating by radiation. The equilibrium temperature of the top copper block was recorded for several values of power dissipated in the resistor while the refrigerator base plate was stabilized at 4 K. From these measurements, the integrated thermal conductivity (3) between 4 K and T was computed. The results of the measurements compared with the theoretical prediction are presented in Figure 2

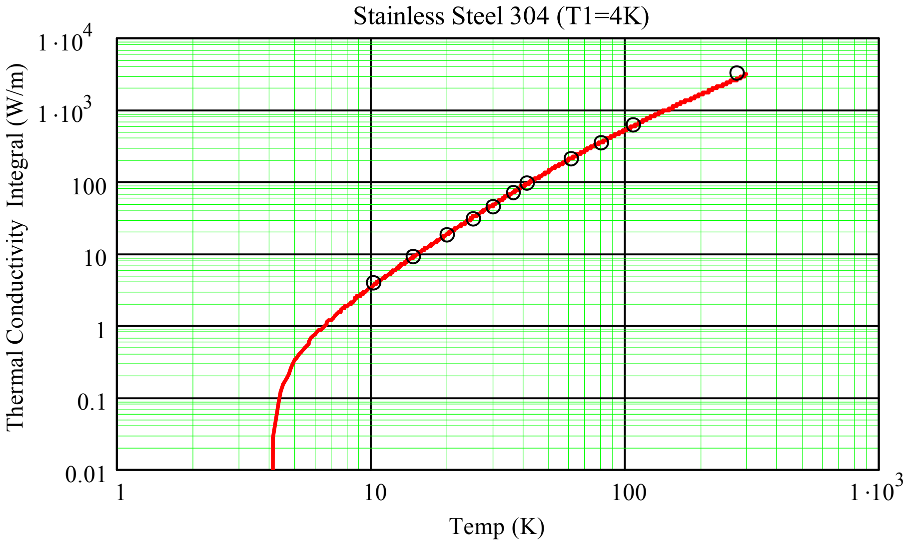

Figure 2: Comparison of the measured Thermal Conductivity Integral measured in a short Q band waveguide piece (black dots) with the calculation from integration of the NIST data for Stainless Steel 304 (red). Note that reference temperature for the integral is 4 K in this plot.

The agreement of experimental and predicted conductivities is very good, confirming the accuracy and utility of the NIST polynomial and the numerical integration for practical cases.

Table I shows the results of the thermal power transmitted by conduction calculated using relation (2) for several waveguide sizes and for some particular values of temperature of interest in cryogenic receivers. Note that the power is quite low especially for the case of low cryogenic temperatures. This makes possible to use short section of waveguide to reduce the microwave loss.

TABLE I

Calculated thermal power conduction for 50 mm and 1 m length thin wall stainless steel waveguide pieces (WR28, WR22 and WR10).

			WR28 26.5-40 GHz	**WR22** 33-50 GHz	**WR10** 75-110 GHz
Inner dimensions and thickness (mm)		a	7.112	5.690	2.540
		b	3.556	2.845	1.270
		t	0.254	0.254	0.254
POWER (mW) L = 50 mm	TEMPERATURE RANGE (K)	4-15	1.077	0.871	0.416
		4-50	15.88	12.84	6.135
		4-300	344.1	278.4	133.0
		15-50	14.80	11.98	5.719
		15-300	343.1	277.6	132.6
		50-300	328.3	265.6	126.8
POWER (mW) L = 1 m		4-15	0.054	0.044	0.021
		4-50	0.794	0.642	0.307
		4-300	17.20	13.92	6.648
		15-50	0.740	0.599	0.286
		15-300	17.15	13.88	6.627
		50-300	16.41	13.28	6.342

Electrical attenuation

The theoretical attenuation (in dB/m) of an empty waveguide (no dielectric inside) due to the finite conductivity of the walls for the dominant TE10 mode can be calculated as [3]:

$$\alpha = 8.686 \cdot \frac{R_s}{\eta \cdot b} \frac{\left(1 + \frac{2 \cdot b}{a} \frac{f_c^2}{f^2}\right)}{\sqrt{1 - \frac{f_c^2}{f^2}}} \frac{dB}{m} \tag{5}$$

Where a and b are the inner dimensions of the waveguide (a>b), R_s is the surface resistance of the metal, η is the dielectric medium impedance, f_c is the cutoff frequency and f the frequency. η, f_c and R_s can be obtained as:

$$\begin{aligned} \eta &= \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.7\Omega \\ f_c &= \frac{c}{2 \cdot a} \\ R_s &= \sqrt{\frac{2 \cdot \pi \cdot f \cdot \mu_0 \cdot \mu_r}{2 \cdot \sigma}} \end{aligned} \tag{6}$$

Where μ_0 and ε_0 are the vacuum permeability and permittivity, c is the speed of light μ_r the relative permeability of the metal and σ its conductivity. Note that SS304 is an austenitic alloy and it is not magnetic ($\mu_r \approx 1$). In the case of magnetic steels the losses are much larger.

Table II presents the values of the DC conductivity of SS304 at ambient and cryogenic temperature (data from [4]). Cryogenic DC conductivity data of copper is not given since it depends strongly on its purity and it is usually characterized by the Residual Resistivity Ratio (RRR) [6]. Note that the value of the SS304 conductivity increases by a factor of ~1.5 when cooled. This implies a reduction of the attenuation by a factor of ~1.2 (in dB) at cryogenic temperature. From the conductivity data at ambient temperature it can be deduced that the theoretical loss of a SS 304 waveguide is a factor of 6.5 (in dB) larger than the loss of a copper waveguide of the same size. The microwave loss of copper at cryogenic temperature can also be affected by the anomalous skin effect [6]. Typically the Cu loss is reduced by a factor of ~3 for common purity material upon cooling from ambient to 4 K.

TABLE II

Bulk DC conductivity of stainless steel 304 at various temperatures compared with copper at ambient temperature.

Temp. .(K)	σ SS 304 (Ω^{-1} m^{-1})	σ Cu OHFC (Ω^{-1} m^{-1})
4	2.06×10^6	
15	2.06×10^6	
50	2.06×10^6	
300	1.39×10^6	5.96×10^7

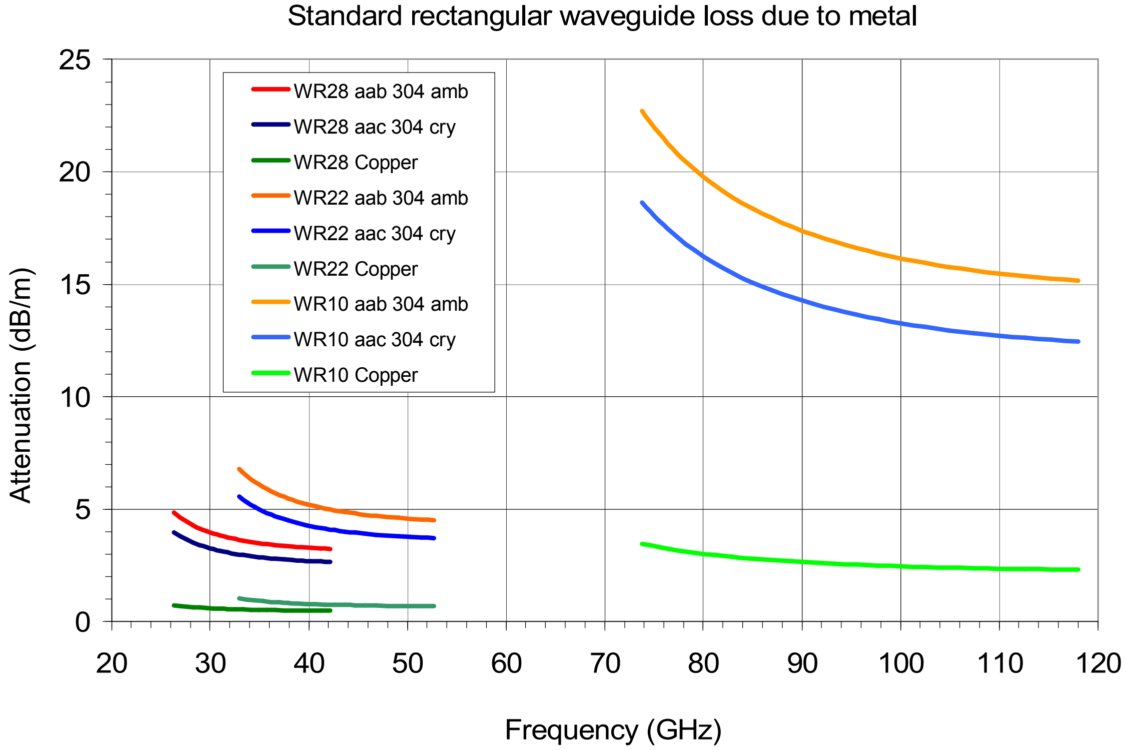

Figure 3: Theoretical loss for WR28, WR22 and WR10 waveguides of stainless steel 304 (ambient and cryogenic temperature) and copper (ambient temperature only).

TABLE III

Theoretical insertion los for 50 mm and 1 m length thin wall stainless steel waveguide pieces (WR28, WR22 and WR10).

			WR28		WR22		WR10	
			26.5 GHz	40 GHz	33 GHz	50 GHz	75 GHz	110 GHz
Inner dimensions (mm)		a	7.112		5.690		2.540	
		b	3.556		2.845		1.270	
Loss (dB) L = 50 mm	TEMP.	CRYO	0.20	0.14	0.28	0.19	0.90	0.64
		AMBIENT	0.24	0.16	0.34	0.23	1.10	0.77
Loss (dB) L = 1 m		CRYO	3.93	2.70	5.54	3.77	18.05	12.71
		AMBIENT	4.79	3.28	6.75	4.59	21.98	15.48

Figures 4, 5 and 6 show the comparison of the theoretically calculated loss for SS304 with experimental measurements obtained with SNA. In general the predictions are too optimistic and an empirical correction factor (~1.1-1.3) should be used for a perfect fit. This is usually justified by the imperfections on the surface of the material. The SNA measurements were obtained using isolators on the detector and generator side to reduce the ripple due to reflections. The “glitches” in the measurements are probably due the effect of undesired harmonics of the waveguide multipliers used.

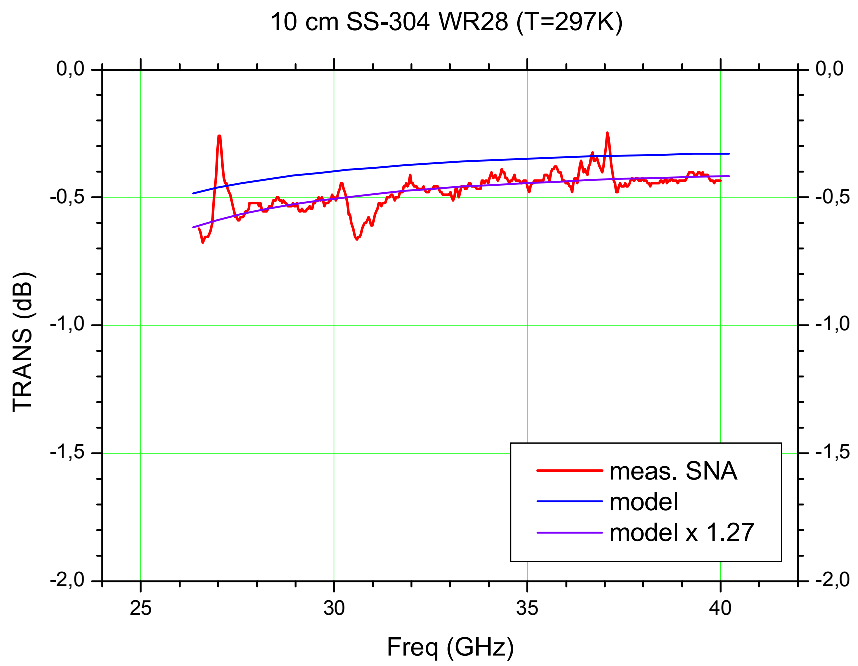

Figure 4: Comparison of the theoretical loss for SS-304 WR28 waveguide at ambient temperature with a Scalar Network Analyzer measurement of a 10 cm piece. Note that a correction factor of 1.27 provides an almost perfect fit.

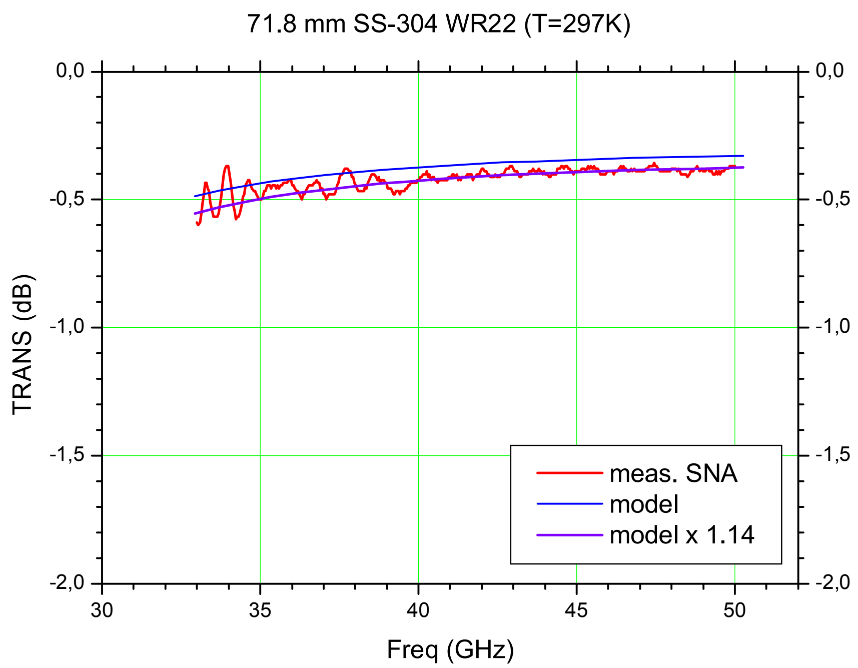

Figure 5: Comparison of the theoretical loss for SS-304 WR22 waveguide at ambient temperature with a Scalar Network Analyzer measurement of a 71.8 mm piece. Note that a correction factor of 1.14 provides an almost perfect fit.

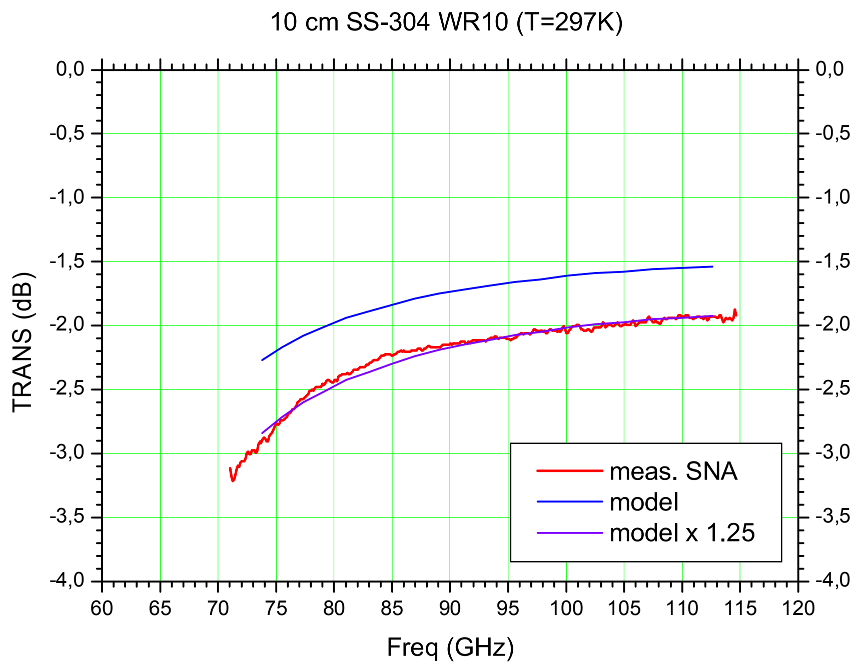

Figure 6: Comparison of the theoretical loss for SS-304 WR10 waveguide at ambient temperature with a Scalar Network Analyzer measurement of a 100 mm piece. Note that a correction factor of 1.25 provides an almost perfect fit.

Application example: heated load

One convenient possibility for the measurement of the noise temperature of a cryogenic amplifier with waveguide input is using a variable temperature heated load. This device is simply a waveguide matched termination fitted with a heating resistor and a temperature sensor. The load can be connected to the amplifier using a short section of thin wall stainless steel waveguide which provides thermal isolation and prevents changing the temperature of the amplifier when the heating resistor is switched on. However, the insertion loss of the waveguide causes some variation in the equivalent noise temperature presented at the input of the amplifier (cooling effect). The equivalent temperature can be easily calculated assuming a linear physical temperature distribution[2] along the waveguide [8]:

$$T_{out} = \frac{T_{load}}{L} + \left(1 - \frac{1 - \frac{1}{L}}{L'}\right) \cdot T_2 - \left[\frac{1}{L} - \frac{1 - \frac{1}{L}}{L'}\right] \cdot T_1 \qquad (7)$$

Where T_{out} is the equivalent noise temperature presented to the amplifier, T_{load} is the equivalent noise temperature at the output of the load, T_2 is the physical temperature on the amplifier side, T_1 is the physical temperature on the load side (usually equal to T_{load}) and L is the power loss (linear). L´ can be calculated as:

[2] A more elaborated solution for the temperature distribution and the noise contribution is presented in appendix II and III.

$$L' = \frac{10 \cdot \log_{10}(L)}{10 \cdot \log_{10}(e)} = \frac{L(dB)}{10 \cdot \log_{10}(e)} \cong 0.23036 \cdot L(dB) \qquad (8)$$

Let assume heated loads for the WR28, WR 22 and WR10 bands connected to the amplifier with 50mm length thin wall stainless steel waveguides. If the amplifier is kept at 15 K and the load is heated to 50 K, the thermal power transfer can be directly obtained from Table I. The effect of the losses can be calculated using (7) with the theoretical loss of Table 3. The results are shown in table IV

TABLE IV

Theoretical equivalent output noise temperature of a heated load at 50 K connected with a thin wall stainless steel waveguide of 50 mm *length to a cryogenic amplifier cooled at 15 K (WR28, WR22 and WR10).*

	WR28		**WR22**		**WR10**	
	26.5 GHz	**40 GHz**	**33 GHz**	**50 GHz**	**75 GHz**	**110 GHz**
Thermal Power dissipated into the amplifier (mW) (50 K to 15 K)	14.80		11.98		5.719	
Equivalent Output Noise Temperature (K)	49.21	49.44	48.90	49.25	46.61	47.54
Equivalent Output Noise Temperature Increment (K)	-0.79	-0.56	-1.10	-0.75	-3.39	-2.46

Conclusions:

Stainless steel thin wall waveguides can be a good option for thermal isolations in cryostats and the thermal power conduction can be kept to very low values at low temperature with relatively short sections. However, care should be taken in keeping the microwave loss low. This could be particularly critical at the higher frequencies were the effect could be particularly noticeable. It will be desirable to develop a good process for quality gold plating the inner part of the stainless steel waveguides in order to reduce the loss for the most demanding applications.

References:

[1] J.E. Jensen, W.A. Tuttle, R.B. Stewart, H. Brechna and A.G. Prodell, "Brookhaven National Laboratory Selected Cryogenic Data Handbook: Thermal Conductivity of Solids (sect. VII)," Brookhven National Laboratory, 1980.

[2] Lake Shore Cryotronics, "Cryogenic Reference Tables," online in: http://www.lakeshore.com/Documents/LSTC_appendixI_l.pdf

[3] Orfanidis, S. J.. "Electromagnetic waves and antennas;" chap. 7, pp. 368-383,. 2002 New Brunswick, NJ: Rutgers University.

[4] C. Y. Ho and T. K. Chu, "Electrical resistivity and thermal conductivity of nine selected AISI stainless steels," pp. 16-17, CINIAS Report 45, Purdue University, September 1977.

[5] J. L. Hesler, A. R. Kerr, and N. Horner, "A Broadband Waveguide Thermal Isolator" ALMA Memo 469, NRAO, 2003.

[6] R. Finger and A. R. Kerr, "Microwave Loss Reduction in Cryogenically Cooled Conductors," Electronics Division Internal Report No. 321, July 3, 2008.

[7] G. A. Ediss, "Measurements and Simulations of Overmoded Waveguide Components at 70-118 GHz, 220-330 GHz and 610-720 GHz," 14th International Symposium on Space Terahertz. Technology, April 2003, Tucson, Arizona.

[8] J. D. Gallego I. López Fernández, "Definition of Measurements of Performance of X Band Cryogenic Amplifiers," Technical Report C.A.Y. 2000-4, July 2000

APPENDIX I:

NIST Data for Stainless Steel 304

	Thermal Conductivity	Specific Heat
UNITS	**W/(m-K)**	**J/(kg-K)**
a	-1.4087	22.0061
b	1.3982	-127.5528
c	0.2543	303.647
d	-0.6260	-381.0098
e	0.2334	274.0328
f	0.4256	-112.9212
g	-0.4658	24.7593
h	0.1650	-2.239153
i	-0.0199	0
data range	4-300	4-300
equation range	1-300	4-300
curve fit % error relative to data	2	5

To view a **thermal conductivity** plot Click here

To view a **specific heat** plot Click here

Curve fit equation of the form:

$$\log_{10} y = a+b(\log_{10}T) + c(\log_{10}T)^2 + d(\log_{10}T)^3 + e(\log_{10}T)^4 + f(\log_{10}T)^5 + g(\log_{10}T)^6 + h(\log_{10}T)^7 + i(\log_{10}T)^8$$

Solves as:

$$y = 10^{a+b(\log_{10}T) + c(\log_{10}T)^2 + d(\log_{10}T)^3 + e(\log_{10}T)^4 + f(\log_{10}T)^5 + g(\log_{10}T)^6 + h(\log_{10}T)^7 + i(\log_{10}T)^8}$$

Where: Coefficients a - *i* are summarized in the appropriate table and T is the temperature in K (x-axis), and y is the property to solve for.

APPENDIX II:

Temperature distribution along the waveguide

Usually the temperature distribution along a waveguide connecting components at different temperatures is assumed to be linear for simplicity. However, this assumption is only valid in the case of constant thermal conductivity with temperature. As this condition is clearly not fulfilled (see figure 3), it is interesting to evaluate the impact of the variability on the real temperature distribution.

The basic equation for the calculation is the fundamental Fourier law thermal conduction from (1), which can be written in the form:

$$\dot{Q} = -k \cdot A \cdot \frac{dT}{dx} \tag{9}$$

Where x is in the axis of the waveguide and the other symbols have the same meaning as in (1). In the steady stage regime the power is constant and does not depend on the x position. Taking the derivative on both sides of (9):

$$\frac{d\dot{Q}}{dx} = \frac{d}{dx}\left(-k \cdot A \cdot \frac{dT}{dx}\right) = 0 \tag{10}$$

Expanding the derivative and assuming that A is constant:

$$\frac{d}{dx}\left(k \cdot \frac{dT}{dx}\right) = \frac{dk}{dx} \cdot \frac{dT}{dx} + k \cdot \frac{d^2T}{dx^2} \tag{11}$$

In the general case k is a function of temperature. Then:

$$\frac{dk}{dT} \cdot \frac{dT}{dx} \cdot \frac{dT}{dx} + k \cdot \frac{d^2T}{dx^2} = \frac{dk}{dT} \cdot \left(\frac{dT}{dx}\right)^2 + k \cdot \frac{d^2T}{dx^2} = 0 \tag{12}$$

Rearranging conveniently and adding the boundary conditions:

$$\begin{aligned} &\frac{d^2T}{dx^2} + \frac{1}{k}\frac{dk}{dT} \cdot \left(\frac{dT}{dx}\right)^2 = 0 \\ &x = 0 \rightarrow T = T_1 \\ &x = L \rightarrow T = T_2 \end{aligned} \tag{13}$$

This is the differential equation with the boundary conditions which must be solved to determine the temperature distribution. Note that if k is constant this reduces to the 1-dimension Laplace equation and the solution is simply a linear interpolation between T_1 and T_2. In the general case is necessary to include k and its derivative. Equation (13) can be solved numerically[3] using the function for k given in appendix I and its derivative which can be easily calculated analytically. One example of the results obtained with the numerical

[3] The solution to the equation was obtained with the function Odesolve of Math CAD v2001i.

calculation for the case of stainless steel conductivity compared with a simple linear interpolation is presented in figure 6.

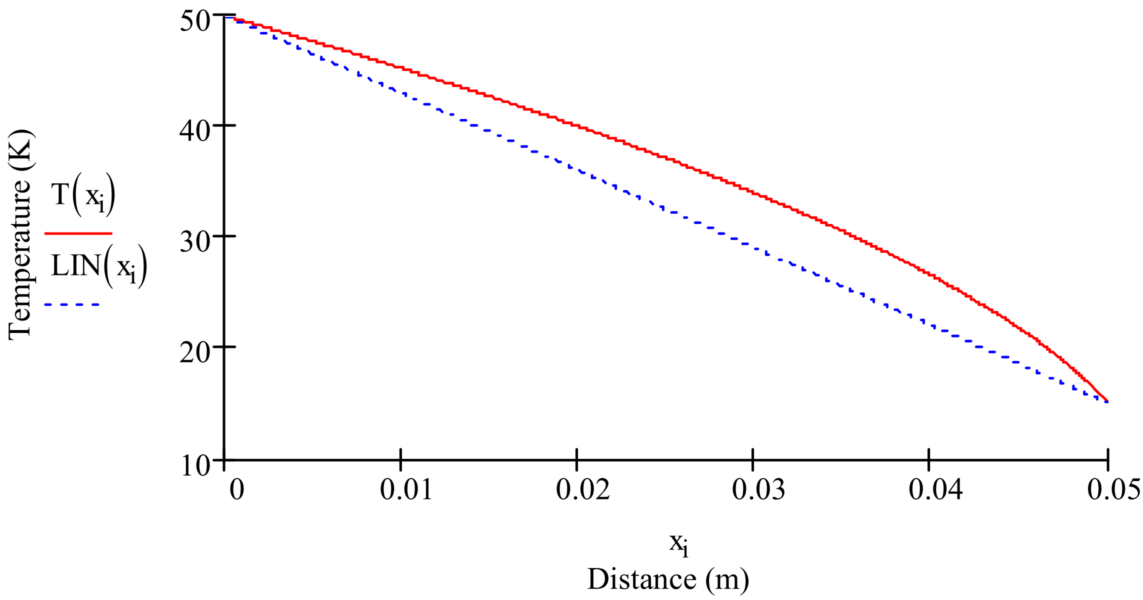

Figure 7: Numerical solution of the differential equation (13) for the temperature distribution of a stainless steel waveguide with the boundary conditions $T_1 = 50$ K and $T_2 = 15$ K compared with a perfectly linear temperature distribution. The length of the waveguide in this example is 50 mm, but the values in the x axis can be changed to any others and the same solution will be valid.

One important property of this differential equation is that the same solution applies to any scaling of the x axis (if the temperatures of the boundary condition are kept constant). So, for example, for a 1 m waveguide the graph of figure 6 will be valid just changing the scale of the x axis to 0-1m. The departure of the real solution from linearity is more evident for low cryogenic temperatures since then the variation of the thermal conductivity (derivative of k) is larger. Note that the values of the temperature of the numerical solution are larger that the values of the linear interpolation for any position x. The maximum departure from linearity in this particular example is obtained at about 1/3 of the distance from the coldest side and it is of ~5 K.

APPENDIX III:

Noise temperature contribution for a non-linear physical temperature distribution.

The calculation of the contribution of the losses in the stainless steel waveguide to the equivalent noise temperature of a heated load is complicated by the fact that the real temperature distribution along the waveguide departs from a perfectly linear interpolation a shown in appendix II. The noise contribution of a perfect attenuator at a uniform physical temperature can be calculated as:

$$T_{out} = \frac{T_{load}}{L} + \left(1 - \frac{1}{L}\right) \cdot T_{ph} \qquad (14)$$

Where T_{out} is the equivalent noise temperature presented to the amplifier, T_{load} is the equivalent noise temperature at the output of the heated load, T_{ph} is the physical temperature of the attenuator and L is the power loss (linear).

As the solution to the differential equation of appendix I was obtained numerically, the result is a set of pairs of points (position and physical temperature). The most efficient method for calculating the noise contribution in this case is to consider the waveguide as a cascade of small attenuators (segments), each one with uniform loss and physical temperature. In this case the loss is considered constant with temperature (good approximation for SS 304), but the same method can be easily extended for variable loss. The loss of each segment is calculated simply as the total loss (in dB) divided by the number of segments and its physical temperature is set to the average of the temperatures at the edges. The formula for the iteration is:

$$\begin{aligned} Tn_0 &= T_{load} \\ Tn_i &= \frac{Tn_{i-1}}{L} + \left(1 - \frac{1}{L}\right) \cdot \left(\frac{Tph_{i-1} + Tph_i}{2}\right) \end{aligned} \qquad (15)$$

Where Tn_i is the equivalent noise temperature after the i segment, Tph_i the physical temperature at point i and L the loss of the segment. L is calculated as:

$$\begin{aligned} L_{dB} &= \frac{Ltotal_{dB}}{m} \\ L &= 10^{\left(L_{dB}/10\right)} \end{aligned} \qquad (16)$$

Where $Ltotal_{dB}$ is the total loss of the waveguide and m the total number of points of the solution of the differential equation (typically 1000). The equivalent noise temperature at the output of the waveguide is Tn_m. The results of the iteration obtained for the example of the solution of figure 6 are presented in table V and figure 7.

TABLE V

*Comparison of the results of the theoretical equivalent output noise temperature of a heated load at **50 K** connected to an amplifier at **15 K** with a SS 304 waveguide with a total loss of **0.64 dB** (as WR10 @ 110 GHz) using different physical temperature distributions. The most accurate estimation is the one obtained with the solution of the differential equation.*

Model of physical temperature distribution	Output Noise Temperature (K)	Correction (K) (T_{load} = 50 K)
Non-linear (differential equation solution)	48.00	**-2.00**
Linear approximation distribution	47.54	**-2.46**
Constant temperature (average of both sides)	47.60	**-2.40**

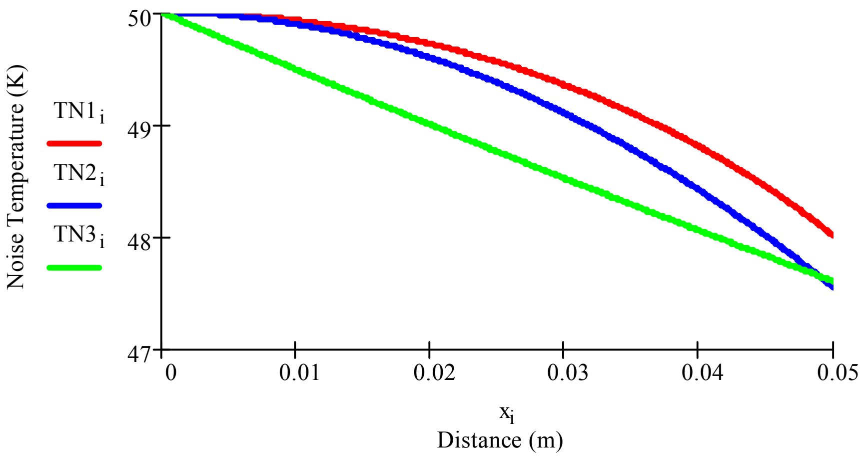

Figure 8: Evolution of the equivalent noise temperature along the waveguide of the example obtained with the three different distributions. Red is for the differential equation solution, blue for the linear approximation and green for the constant temperature (average of both ends).

Note that the effect in the correction of the real temperature distribution in this case is smaller than the value obtained with the approximation of the linear distribution. The error in the correction of the linear approximation in this example is ~0.5 K, which is ~25%. It is interesting to note that the results obtained with the linear temperature distribution and the constant temperatures (average of both ends) are remarkably similar.