

**The atmosphere in the 40m RT environment.  
Water vapour and opacity**

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## Revision history

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## 1 Introduction

The radio window covered by the receivers in Yebes goes from 2 GHz to 115 GHz. In this interval opacity comes mainly from water vapour and oxygen. We have investigated the water vapour content along 10 years in the environment of the 40 m radiotelescope. Our purpose is to estimate which is the most suitable epoch of the year for 3 mm observations. Estimations have been made indirectly, based on weather parameters in the surface of the observatory since direct measurements with a water vapor radiometer are not available.

The opacity of the atmosphere can be estimated from the local water vapour content assuming a multilayer standard atmosphere. This estimation is usually done by two software models: ATM (Pardo 2001) and AM (Payne 2004). Both models take into account contributions from dry air ( $N_2$ ,  $O_2$ ,  $O_3$ ) and  $H_2O$ .

## 2 Water vapour content in the atmosphere

The amount of precipitable water can be roughly estimated from weather conditions in the surface. Most part of the derivation below is shown in Butler (1998).

The mass of precipitable water ( $m_{lw}$ ) in the atmosphere can be written as:

$$m_{lw} = \rho_l A h \quad (1)$$

where  $\rho_l$  is the density of liquid water,  $A$  is the section of a column in the atmosphere, and  $h$  the height of the water column.

The water column can be estimated by integrating the number of molecules along a column through the atmosphere, starting at the level of the observatory ( $Z_s$ ). Let  $m_{wv}$  the mass of water vapour:

$$m_{wv} = A m_{H_2O} \int_{Z_s}^{\infty} n_{wv} dZ \quad (2)$$

where  $m_{wv}$  is the mass of the water vapour column of section  $A$ ,  $m_{H_2O}$  the mass of a molecule of water (18 uam) and  $n_{wv}$  the density number of water vapour molecules.  $Z$  is the height along the atmosphere of Earth. The density number of molecules of water vapor may be considered to follow an exponential distribution as a function of height ( $Z$ ):

$$n_{wv}(Z) = n_s e^{-(Z-Z_s)/H}$$

where  $n_s$  is the number of water vapour molecules at the surface of the Earth,  $Z_s$  the height of the observatory and  $H$  is a scale factor that indicates the height at which the column of water is 0.37 times that at the local surface. When  $Z - Z_s = H$ , the number of water vapour molecules is  $n_{wv} = 0.37 n_s$ . This expression will only be valid in the troposphere, where water vapour is present (below 14 km).

Integrating we get:

$$m_{wv} = A m_{H_2O} n_s H$$

We can equal equations 1 and 2 and we get:

$$\rho_l A h = A m_{H_2O} n_s H$$

and the column of precipitable water is:

$$h = \frac{m_{H_2O} n_s H}{\rho_l} \quad (3)$$

According to the law of perfect gases:

$$n_s = \frac{P_{H_2O}}{K T_s}$$

where  $K$  is the Boltzmann constant and  $T_s$  the temperature in Kelvin at the surface of the Earth in the observatory. The partial pressure of water,  $P_{H_2O}$  in mbar can be written as a function of the dew temperature,  $T_d(^{\circ}C)$  (Clark 1987):

$$P_{H_2O} = e^{1.81+17.27 \frac{T_d(^{\circ}C)}{T_d(^{\circ}C)+237.7}}$$

The dew point is the temperature to which the air must be cooled, at barometric pressure, for the water vapour to condense into water. A well known approximation is the following:

$$T_d(^{\circ}C) = \frac{b \gamma}{a - \gamma}$$

where

$$\gamma = a \frac{T_s(^{\circ}C)}{b + T_s(^{\circ}C)} + \ln(hr/100)$$

and where  $T_s(^{\circ}C)$  is the ambient temperature in Celsius,  $hr$  is the relative humidity,  $a = 17.271$ ,  $b = 237.7$  and  $T_d(^{\circ}C)$  is the dew temperature in Celsius. NOAA uses Bolton (1980) values  $a = 17.67$  and  $b = 243.5$  which provides an accuracy of 99.9% for temperatures between -35 C and 35 C and relative humidity between 1 and 100%. We will use the first set of values for  $a$  and  $b$ .

The partial pressure of water in mbar can be written then as:

$$P_{H_2O} = e^{1.81} e^{a \frac{T_d(^{\circ}C)}{T_d(^{\circ}C)+237.7}} \quad (4)$$

$$= 6.11 \frac{hr}{100} e^{17.271 \frac{T_s(^{\circ}C)}{237.7+T_s(^{\circ}C)}} \quad (5)$$

Planesas (1987) uses a similar function for the partial vapour pressure (mbar):

$$P_{H_2O} = 6.015 \frac{hr}{100} \left( \frac{T_s(^{\circ}C)}{273} \right)^{-5.31} e^{25.22 \frac{T_s(^{\circ}C)-273}{T_s(^{\circ}C)}} \quad (6)$$

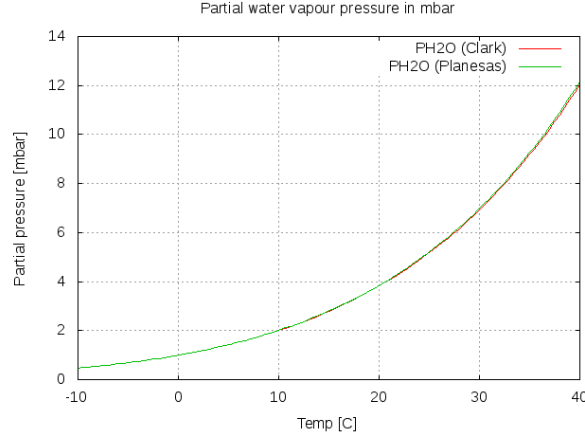


Figure 1: Water vapor partial pressure as a function of the ambient temperature according to equations 5 and 6.

We have compared equations 5 and 6 in Fig. 1 as a function of the ambient temperature. The difference between both expressions is negligible. We have been using expression 6 since 2010 in the 40 m radiotelescope.

Hence the column of precipitable water vapour depends on a scale factor ( $H$ ), the relative humidity ( $hr$ ) and the surface atmospheric temperature ( $T_s(^{\circ}C)$ ). Replacing in 3 we get:

$$h = \frac{6.11 \frac{hr}{100} 100 e^{17.271 \frac{T_s(^{\circ}C)}{237.7+T_s(^{\circ}C)}} m_{H_2O}}{\rho_l K T_s} H$$

where we have multiplied by 100 to convert from mbar to Pa.

For practical purposes, and after replacing the values of each parameter in the proper units, the column of precipitable water in mm may be written as:

$$h[mm] = 6.11 \cdot 10^5 \frac{hr}{100} \frac{e^{1.81+17.27 T_s(^{\circ}C)/(T_s(^{\circ}C)+237.7)} 1.66053886 \cdot 10^{-27} 18}{10^3 \cdot 1.3806503 \cdot 10^{-23} (T_s(^{\circ}C) + 273)} H \quad (7)$$

$$= 1.3227 \cdot 10^{-2} \frac{e^{17.27 T_s(^{\circ}C)/(T_s(^{\circ}C)+237.7)}}{T_s(^{\circ}C) + 273} hr H \quad (8)$$

where  $H$  is in meters and  $hr$  is the relative humidity (values range from 0 to 100)

### 3 The height scale factor

According to Ulich (1980),  $H$ , the scale factor is between 1500 and 2000 m. We have investigated this scale from data obtained by the balloons released by Barajas airport everyday. Barajas is at a straight distance of 45 km approximately and releases two balloons per day which provide data for pressure, height, temperature, dew point, relative humidity and other parameters along its upwards trajectory through the atmosphere from 630 m to 30 km. These values allow to determine the number of molecules of water as a function of height:

$$n_{H_2O} = \frac{P_{H_2O}}{K T_s} = \frac{e^{1.81} e^{17.271 \frac{T_d(^{\circ}C)}{T_d(^{\circ}C)+237.7}}}{K T_s}$$

If we represent  $n_{H_2O}$  as a function of height we should get the following dependency:

$$n_{H_2O} = n_{H_2O}(0)e^{-Z/H}$$

or in a logarithmic scale:

$$\log n_{H_2O} = -Z \log n_{H_2O}(0) \frac{1}{H}$$

where  $n_{H_2O}(0)$  is the density of water vapour at the surface. The slope of the line is the inverse of the height scale.

Data from Barajas airport can be retrieved from the University of Wyoming web page on sounding measurements (<http://weather.uwyo.edu/upperair/sounding.html>). The code for Barajas airport is LEMD. Fig. 6 shows the density of water vapour as a function of height for 4 days in different seasons of years 2011 and 2012. Data points approximately lay along a straight line, as expected, and the determined scale factor ranges between 1180 to 1453 m.

In order to obtain a usable scale factor we have averaged the data for a whole year, starting on may 2011 and ending in April 2012 from a total of 654 series of data (2 series per day). We have also plotted the averaged values per month in Fig. 3. The average scale factor for the last year is  $1340 \pm 120$ . This value will be used from now on in the 40 m radiotelescope atmosphere model, replacing the 2500 m that has been used for the last 2 years since 2010.

## 4 The ammount of precipitable water along the year

We have analyzed data from our weather station since 2002 and computed the ammount of precipitable water using equation 8. Fig. 8 shows the results for the last 10 years.

According to Fig. 8, the water vapour ranges from 3 mm to 20 mm. There is a periodic behaviour, with minimum values in winter and maximum values in summer. In order to investigate with greater time resolution we show the behaviour for year 2011 in Fig. 5. According to this figure the best months for 3 mm observations are december, january, february and march; during all these months the column of water vapour is almost always below 10 mm. The best values were in the last half of january and first halt of february, matching the lowest temperatures and relative humidity in the whole year.

Fig. 6 shows the behaviour the water vapour content for January and July 2011. There is a periodic daily behaviour associated to the temperature of the environment. The lower the temperature the smaller the water vapour content.

We can conclude that, according to water vapour content in the atmosphere, 3 mm observations should be restricted to winter. Winter nights are the optimum time at which this observations should be performed.

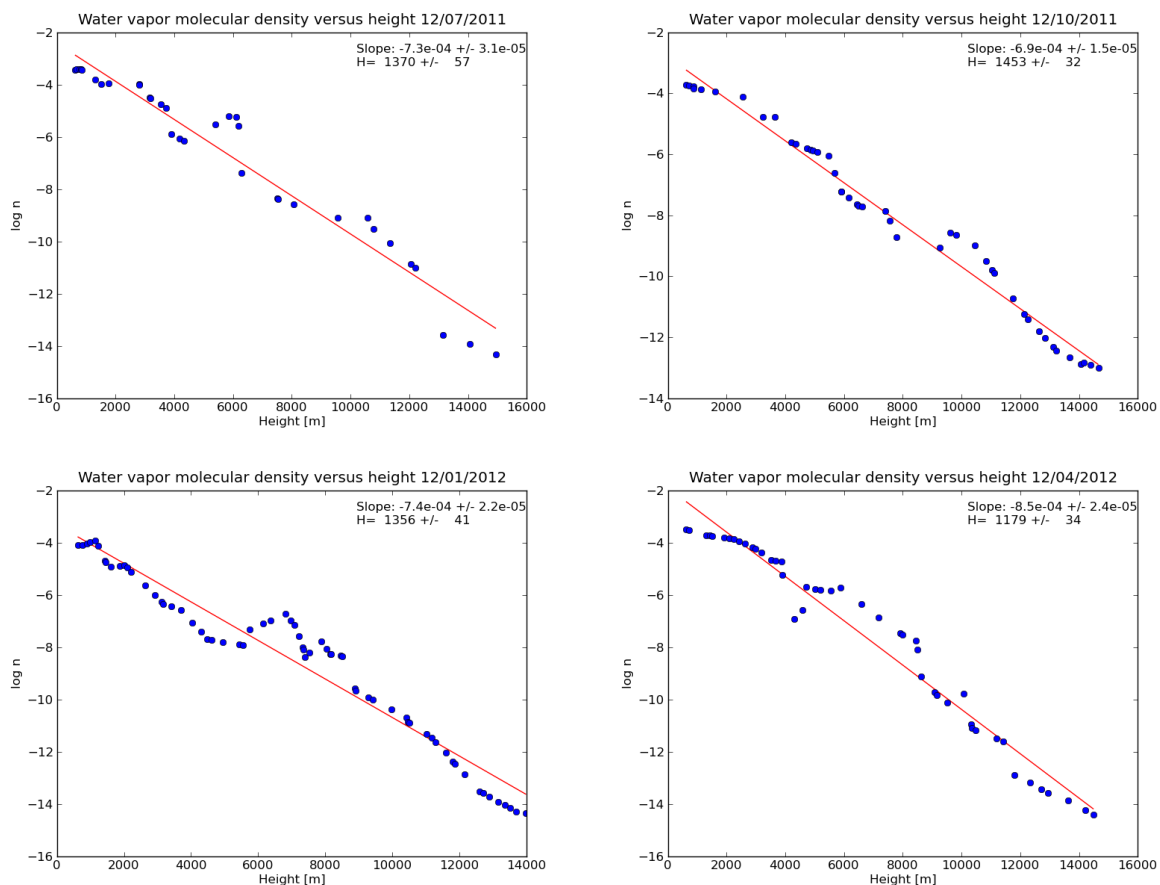


Figure 2: Water vapor density as a function of height for four different dates: 12/07/2011, 12/10/2011, 12/01/2012, 12/04/2012.

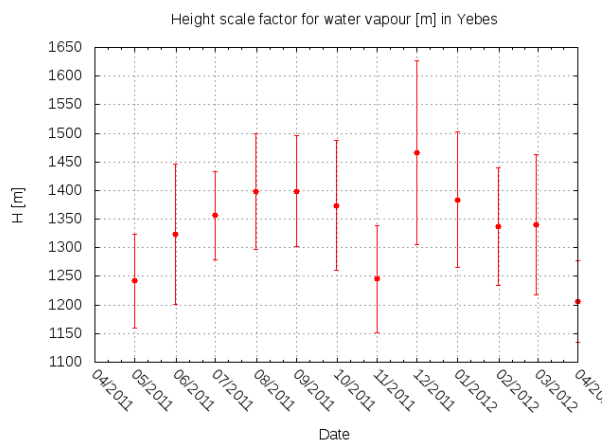


Figure 3:  $H$  scale factor for different months of the year. The value for each month is an average of all days in that month.



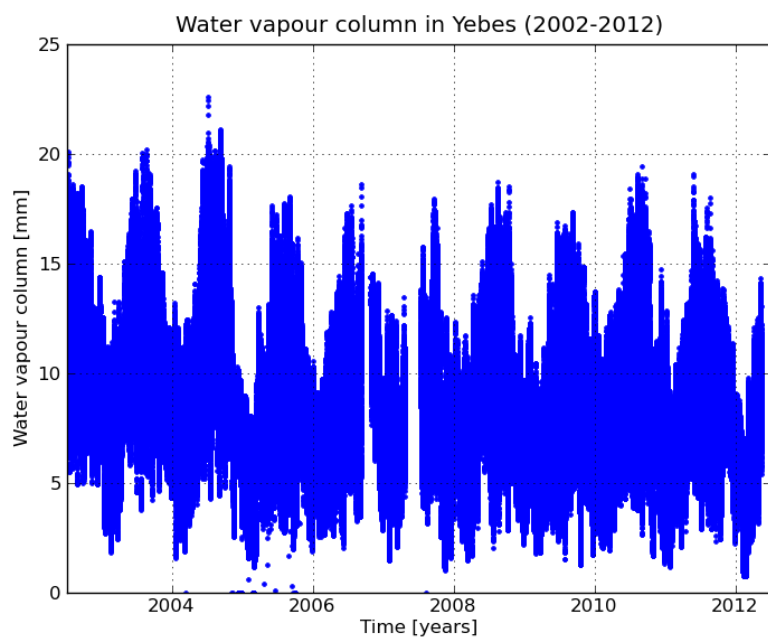


Figure 4: Water vapour for the last 10 years in Yebes. Data obtained from weather parameters in the environment

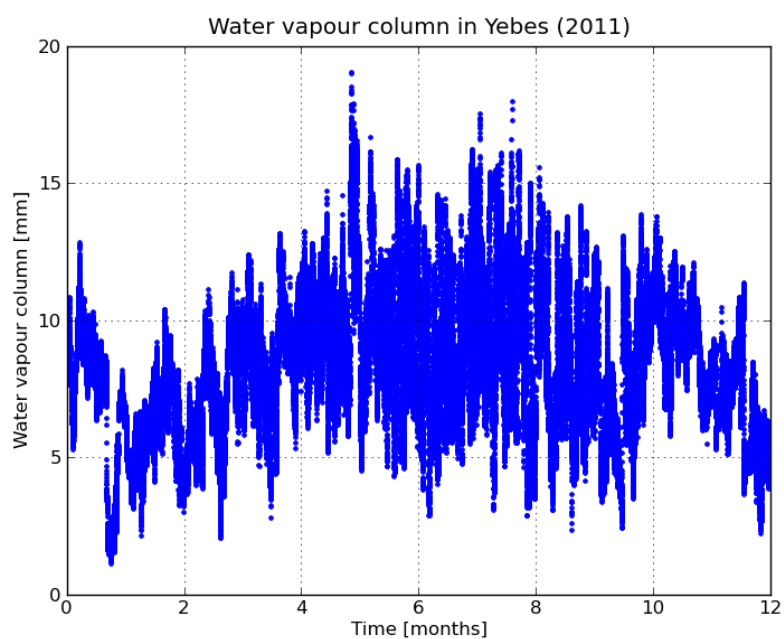


Figure 5: Water vapour for year 2011.

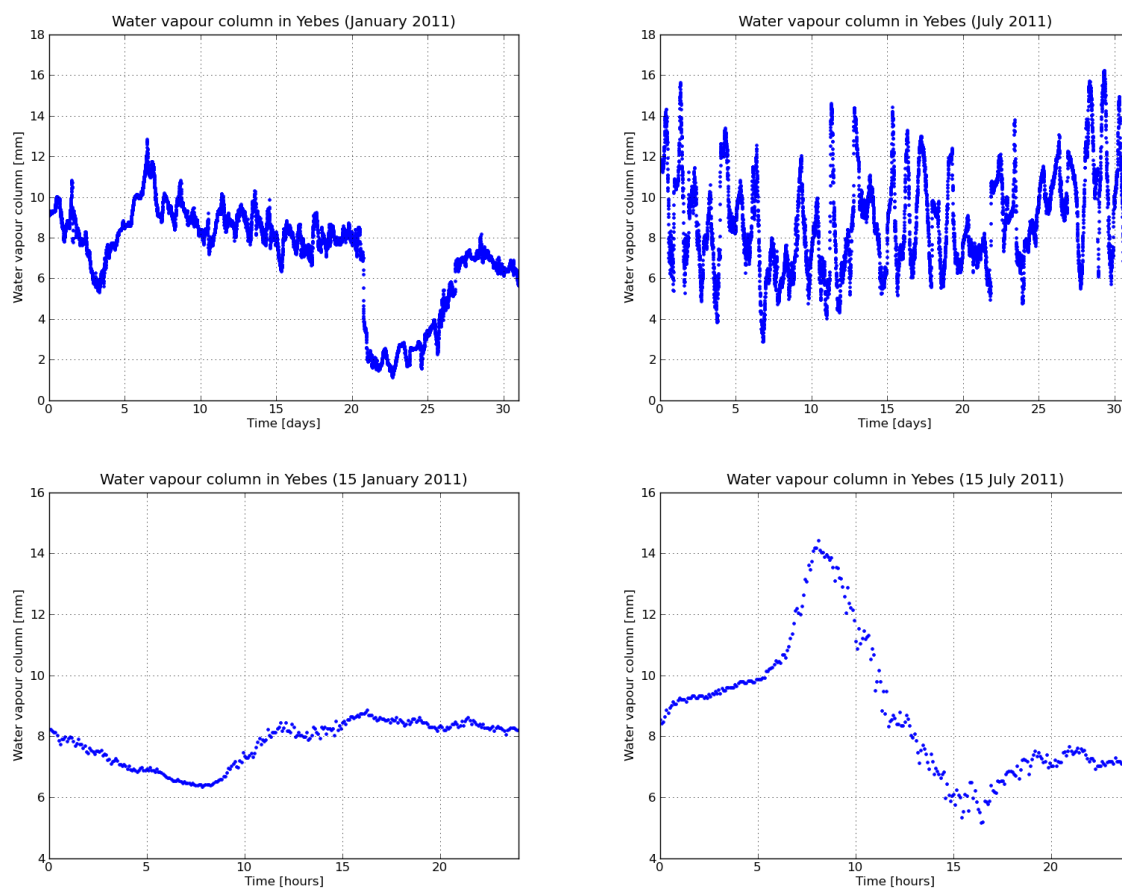


Figure 6: Top panels: column of water vapor in winter (January 2011) and summer (July 2011). Bottom panels: water vapour content during 24 hours in one day of each month.

## 5 A word on opacity

Opacity in the 22 GHz to 115 GHz radio window strongly depends on water vapour content and oxygen. Estimating the contribution of water vapour to opacity is crucial because it depends strongly on weather conditions, whereas contribution from oxygen or continuum is rather constant and only depends on the height of site.

As seen in previous section, the water vapour content has been overestimated since 2010 due to a wrong scale factor (2500 m). Taking into account the new scale factor is 1340 m, the amount of overestimation is 86% approximately. That implies that the estimation of opacity towards the zenith was also too large. Fig. 7 left shows the dependency of opacity on water vapour as predicted by ATM (Pardo 2001). It can be considered that there is an approximate linear dependency at 22 GHz and 45 GHz, but this dependency is less clear at 88 GHz, specially for high water vapour content.

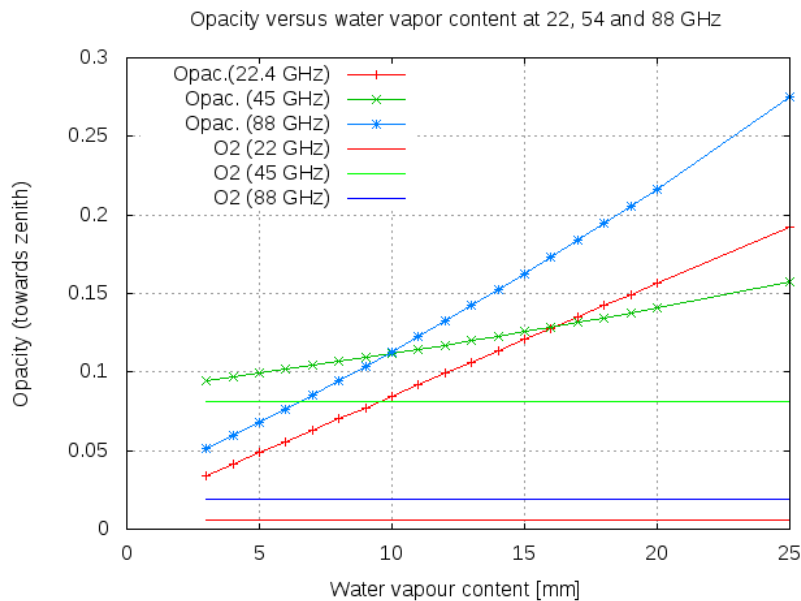


Figure 7: Zenith opacity versus water vapour content for 22.4, 45 and 88 GHz. The horizontal lines indicate the oxygen line contribution to opacity.

The change of opacity due to the water vapour content can be estimated from the slope of the curves in Fig. 7: 0.0075/mm at 22 GHz, 0.0025/mm at 45 GHz and 0.01/mm at 88 GHz. The dependence on weather conditions is smaller at 45 GHz than at 22 or 88 GHz. However the overall opacity at 45 GHz is larger than at 88 GHz, due to the contribution of the oxygen line, whose content can be considered approximately constant for different weather conditions.

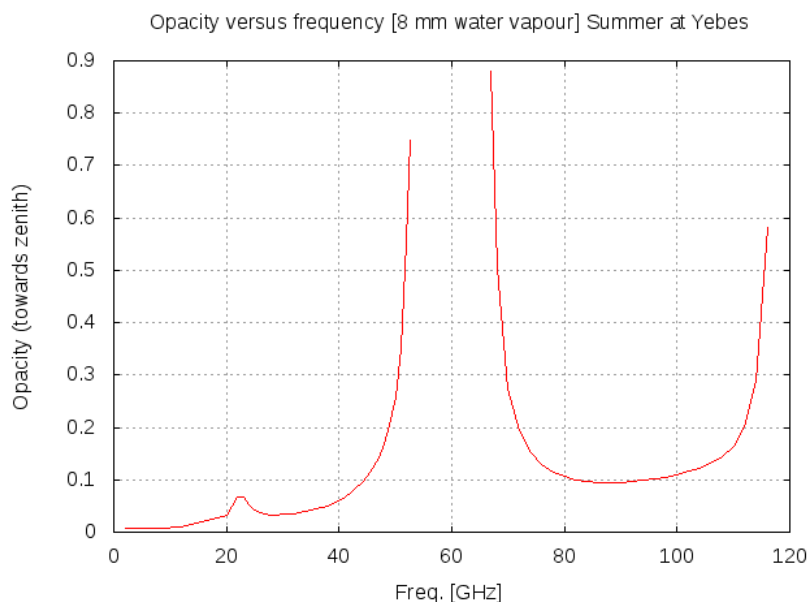


Figure 8: *Opacity versus frequency assuming a standard atmosphere in summer at Yebes for a water vapour content of 8 mm. The small line at 22 GHz is the water vapour line. The broad lines at 60 and 120 GHz are due to oxygen in the atmosphere.*

## References

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