Vertical deformation of the counterweights in the 40m radiotelescope.

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1 Introduction.

The counterweights of a radiotelescope are the structure in charge of compensating the weight of the parabolic surface and its backup structure. The purpose is the reduction of the torque that the servomotors have to generate while moving in elevation. They are a metallic structure with a ballast at the free end and connected to the backup structure in the opposite side. Between both ends the counterweights are fixed to the fork through the elevation bearings. In the case of the 40m radiotelescope of Yebes, the counterweight is a prismatic box with a complex structure inside to achieve the required stiffness.

Any structure is exposed to ambient temperature changes and to mechanical strains due to the forces it supports. Both effects produce a deformation of the structure. On one side, the change of the external temperature produces a change in the dimensions. On the other side, the forces on the structure create internal tensions that, due to the elasticity of the materials, produce a deformation of its shape.

This report defines a model for the static deformation in the vertical plane of the counterweight structure using the classical theory of strength of materials. This deformation is principally caused by changes of temperature and by gravity. Action of wind and tensions generated by the connections with the elevation bearing and the backup structure of the antenna are ignored. The transient deformation due to changes of acceleration in both axes is not the scope of this report

The main application of this study is to assess the amplitude of the systematic relative movement affecting the survey observations of targets located at the free end of the counterweights. The observation of these targets, while the radiotelescope is turning, is needed in order to estimate the invariant reference point of the radiotelescope [REF-1]. The counterweights were chosen as they turn similarly, but inversely, to the radiotelescope dish and because they are likely less prone to deformation. Yet, the amplitude and behavior of this deformation needs to be evaluated.

2 Description of the counterweights.

The 40m radiotelescope of Yebes has two identical counterweights, one at each side of the fork. They can be divided in three sections: the ballast carrier, the cantilever arm and the connection structure. The ballast carrier contains the ballast for compensating the weight of the backup structure and parabolic surface. The cantilever arm connects the ballast carrier with the elevation bearing. Finally, the connection structure is the section between the elevation bearing and the backup structure. The connection structure is not in the scope of the study, and only the structure between the elevation bearing and the free end of the counterweight is considered.

The structure of the counterweight is completely made of steel. It consists in a closed prismatic box with a lot of beams, plates and reinforcements inside for strengthening purpose. Apart from the metallic structure, the ballast carrier has a space which contains the ballast fixed to the structure. This ballast is a solidified mixture of concrete and small pieces of steel.

Each counterweight of the radiotelescope can be considered as a structure tilted the elevation angle ξ and fix at the point E where it is anchored to the elevation bearing. The elevation bearing is supposed to be an invariant point, with no movement and no rotation. The mass of both sections are different and too high, specially for the ballast carrier, to neglect the action of gravity. This action can be assumed as vertical and uniform forces along the length of each section. This situation is represented in Figure 1, where section A is the cantilever arm and section B the ballast carrier.

Any cross-section of the counterweight is considered symmetrical with respect a line that halves the section perpendicularly to its longer side. Let define the counterweight axis as the straight line perpendicular to the cross-section at the middle points of that axis of symmetry. The counterweight position is then defined by the counterweight axis, denoted as the dash and point line in Figure 1.

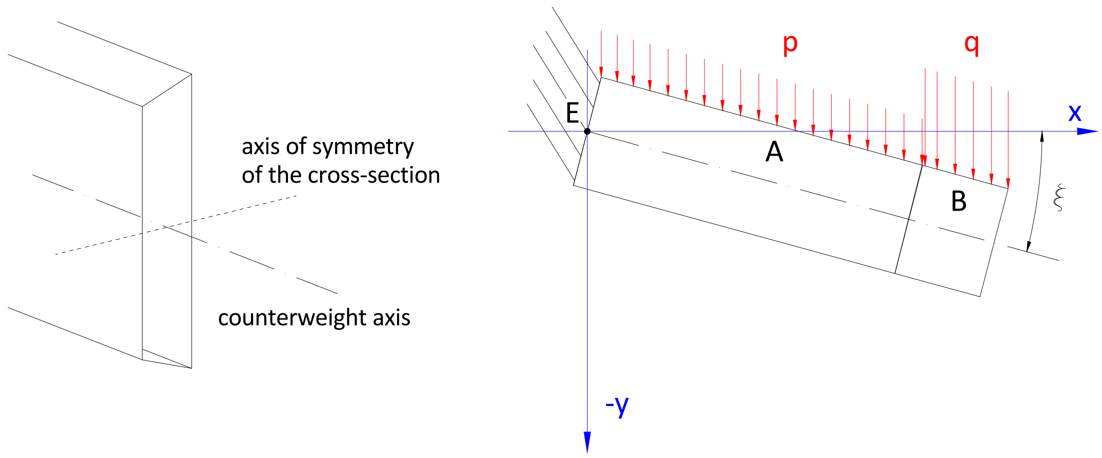


Figure 1. Cantilever simplification.

In Annex I there are two drawings of the counterweight. The first one shows the external appearance and dimensions, and the second one shows the internal structure with some longitudinal cross-sections.

3 Deformation by ambient temperature changes.

Any object experiments a change of its dimensions due to change of its temperature. Let suppose a slowly change of ambient temperature and a uniform heat transmission into the object. Then, a three-dimensional object will experiment a volumetric deformation given by the following expressions:

$$\Delta l_x = \alpha_T \cdot L_x \cdot \Delta T$$
$$\Delta l_y = \alpha_T \cdot L_y \cdot \Delta T$$
$$\Delta l_z = \alpha_T \cdot L_z \cdot \Delta T$$

where L : original length for each axis
α_T : coefficient of linear thermal expansion.
ΔT : difference between current temperature and temperature for the given L.

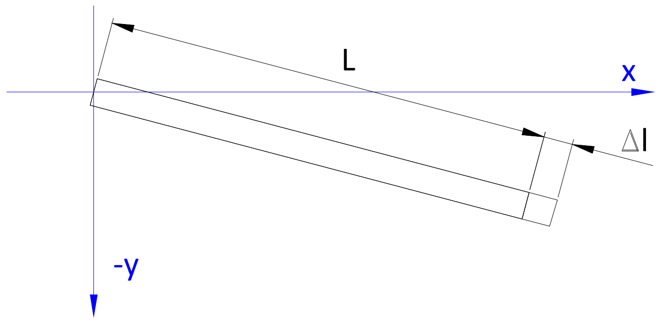


Figure 2. Expansion due to change of ambient temperature.

Assuming the dimensions of the structure for an ambient temperature of 25 ^oC, the change of length by ambient temperature is:

$$\Delta l = \alpha_T \cdot L \cdot (T_{amb} - 25\ ^oC) \tag{1}$$

Only the deformation along the longitudinal axis of the counterweight will be taken into account since it is the only that contributes to the deformation along the counterweight axis. For calculating the deformation at a point out of the counterweight axis, the transverse deformation should be also taken into account.

4 Deformation by gravity.

The action of gravity in any body involves a vertical force towards the surface of the Earth. The intensity of the force depends, among others factors, on its mass. Normally the action of gravity in a structure is neglected compared with the external forces it has to support. In the case of the counterweight, the ballast carriers are designed to compensate the weight of the parabolic structure, and their mass are considerable. The cantilever arms are designed to support the ballast carrier, and that implies a heavy structure. Therefore, the structure of the counterweights only supports the gravitational forces in its own mass.

Moreover, the cantilever arm has to support the ballast carrier. This involves an additional force applied at the end of its length, which can be represented as a pure moment plus a vertical downward force. This means more stress and an increase of the deformation in this section.

To study the deformation in the counterweight, any force under study must be decomposed in the longitudinal and perpendicular axes to the counterweight axis as shown in Figure 3. The longitudinal component produces a tension that elongates the structure, and the perpendicular one produces a shear and a moment that bend the structure. The resultant forces are:

Longitudinal force: $F_L = F \cdot \sin\xi$

Transversal force: $F_T = F \cdot \cos\xi$

Where ξ is the elevation of the antenna.

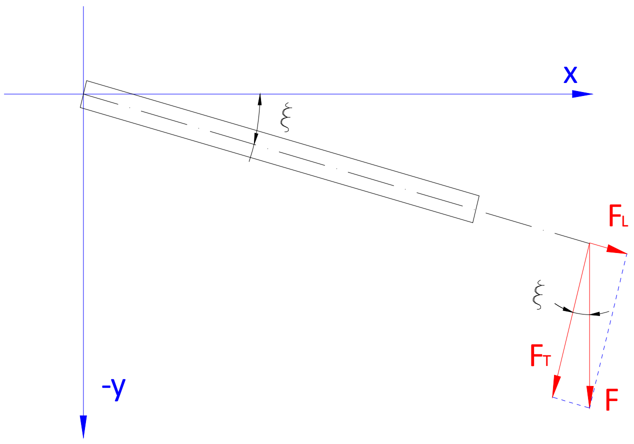


Figure 3. Decomposition of the weight.

4.1 Deformation by tension.

A beam under the action of a normal force to its cross-section is said to be strained. When the force tries to lengthening the beam it is said to be under tension, and when the force tries to shortening the beam it is said to be under compression. In any of these situations the beam experiment a change of its length given by the Hooke's law:

$$\delta = \frac{F \cdot l}{A \cdot E}$$

Where *F* : normal force to the beam
l : length of the beam.
A : area of the cross-section
E : modulus of elasticity of the material

Now, consider a vertical beam with length *l* supported at the upper side. At the bottom the beam support a force *Q*, Any moment applied on the beam is not considered at this moment because it does not create a uniform longitudinal tension. The weight *P* of the beam is too high to be neglected. The situation is shown in Figure 4.

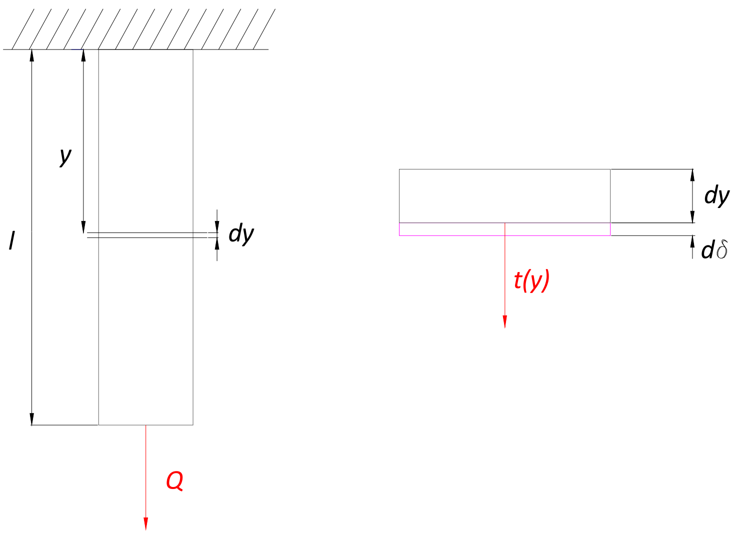


Figure 4. Structure under tension.

A section with thickness *dy* at a distance *y* from the top surface supports a tension force *t(y)* due to *Q* and the weight of the beam behind the section:

$$t(y) = p \cdot (l - y) + Q$$

Where p *:* weight per unit length = *P/l*.

So the elongation *dδ* of the section is:

$$d\delta = \frac{t(y) \cdot dy}{A \cdot E} = \frac{p \cdot (l - y) + Q}{A \cdot E} \cdot dy$$

The total elongation is the sum of all differential elongations:

$$\delta = \int_0^l \frac{p \cdot (l - y) + Q}{A \cdot E} \cdot dy = \frac{1}{A \cdot E} \cdot \left[p \cdot \left(l \cdot y - \frac{y^2}{2} \right) + Q \cdot y \right]_0^l =$$

$$= \frac{1}{A \cdot E} \cdot \left[p \cdot \left(l^2 - \frac{l^2}{2} \right) + Q \cdot l \right] = \frac{1}{A \cdot E} \cdot \left(p \cdot \frac{l^2}{2} + Q \cdot l \right)$$

$$\delta = \frac{l}{A \cdot E} \cdot \left(\frac{P}{2} + Q \right)$$

Taking into account the tilting of the beam as shown in Figure 3:

$$\delta = \frac{l}{A \cdot E} \cdot \left(\frac{P \cdot \sin \xi}{2} + Q \cdot \sin \xi \right) \tag{2}$$

4.2 Deformation by bending.

Perpendicular forces to a beam produce a moment between the point of application and the supports which tries to bend the beam, causing a deformation in the plane of application of the forces. Considering an horizontal beam, the bending in a point *m* of the beam is defined by the angle θ of the tangent at that point and the deflection *f* from the point to the horizontal axis of the beam, as shown in Figure 5. It is assumed that any cross-section of the original beam continues plane after the bending. The bending at a point of the cantilever is defined by the following equations:

$$\theta(m) = \frac{1}{E \cdot I_z} \cdot \int_0^m M(x) \cdot dx \quad [rad]$$

$$df = dx \cdot \sin \theta \Rightarrow f(m) = \int_0^m \sin \theta(x) \cdot dx \approx \int_0^m \theta(x) \cdot dx$$

Where *E* : modulus of elasticity of the material

I_z : moment of inertia for the cross-section in *m*, defined later in page 7.

M(x) : moment supported by the section in *x*.

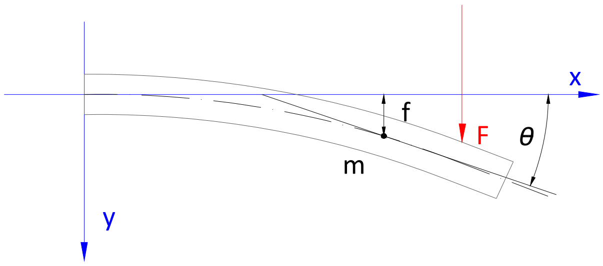


Figure 5. Bending of a beam.

Now let calculate the moment for any section at a distance x of the support. Consider a cantilever supporting a pure moment *M* and a vertical force *Q* at the free end. Consider also that the weight of the cantilever is not depreciable and is assumed as a uniform distributed load along the cantilever of intensity *p* per length unit. This situation is shown in the following drawing:

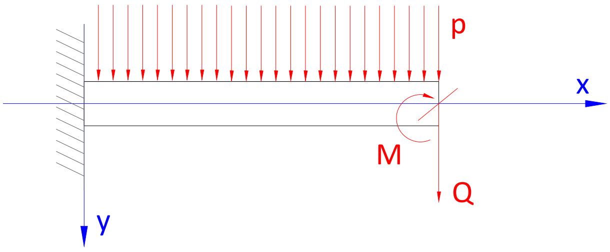


Figure 6. General situation of a cantilever.

The moment at a point *m* is the sum of the moment *M*, the moment due to the force *Q*, and the moment due to the weight from m to the free end of the structure:

$$M(m) = M + Q\cdot(l-m) + \int_m^i x\cdot(p\cdot dx) = M + Q\cdot(l-m) + p\cdot\frac{(l-m)^2}{2}$$

So the angle at a point *x* is:

$$\theta(x) = \frac{1}{E\cdot I_z}\cdot\int M + Q\cdot(l-x) + p\cdot\frac{(l-x)^2}{2}\cdot dx =$$

$$= \frac{1}{E\cdot I_z}\cdot\left[(M+Q\cdot l)\cdot x - Q\cdot\frac{x^2}{2} + \frac{p}{2}\cdot\left(l^2\cdot x - l\cdot x^2 + \frac{x^3}{3}\right)\right]$$

And at the end of the structure is:

$$\theta(l) = \frac{1}{E\cdot I_z}\cdot\left[M\cdot l + Q\cdot l^2 - Q\cdot\frac{l^2}{2} + \frac{p}{2}\cdot\frac{l^3}{3}\right] = \frac{1}{E\cdot I_z}\cdot\left(M\cdot l + Q\cdot\frac{l^2}{2} + p\cdot\frac{l^3}{6}\right)$$

The deflection at the end of the structure is:

$$f(l) = \frac{1}{E\cdot I_z}\cdot\int_0^l (M+Q\cdot l)\cdot x - Q\cdot\frac{x^2}{2} + \frac{p}{2}\cdot\left(l^2\cdot x - l\cdot x^2 + \frac{x^3}{3}\right)\cdot dx =$$

$$= \frac{1}{E\cdot I_z}\cdot\left[(M+Q\cdot l)\cdot\frac{x^2}{2} - Q\cdot\frac{x^3}{6} + \frac{p}{2}\cdot\left(l^2\cdot\frac{x^2}{2} - l\cdot\frac{x^3}{3} + \frac{x^4}{12}\right)\right]_0^l =$$

$$= \frac{1}{E\cdot I_z}\cdot\left[M\cdot\frac{l^2}{2} + Q\cdot\frac{l^3}{2} - Q\cdot\frac{l^3}{6} + \frac{p}{2}\cdot\left(\frac{l^4}{2} - \frac{l^4}{3} + \frac{l^4}{12}\right)\right] = \frac{1}{E\cdot I_z}\cdot\left(M\cdot\frac{l^2}{2} + Q\cdot\frac{l^3}{3} + p\cdot\frac{l^4}{8}\right)$$

Taking into account the tilting of the counterweight as shown in Figure 3, and using degrees instead of radians:

$$\theta(l) = \frac{1}{E\cdot I_z}\cdot\left(M\cdot l + Q\cdot\frac{l^2}{2}\cdot\cos\xi + p\cdot\frac{l^3}{6}\cdot\cos\xi\right)\cdot\frac{180}{\pi}\ [deg] \qquad (3)$$

$$f(l) = \frac{1}{E\cdot I_z}\cdot\left(M\cdot\frac{l^2}{2} + Q\cdot\frac{l^3}{3}\cdot\cos\xi + p\cdot\frac{l^4}{8}\cdot\cos\xi\right) \qquad (4)$$

4.2.1 Moment of inertia.

The moment of inertia is a factor that appears when studying the bending of beams. It depends only on the shape and dimensions of the cross-section of the beam along the longitudinal axis. The moment of inertia from an axis is defined as the sum of any small element of area multiplied by the square of its distance to such axis:

$$I_z = \int y^2 dA \tag{5}$$

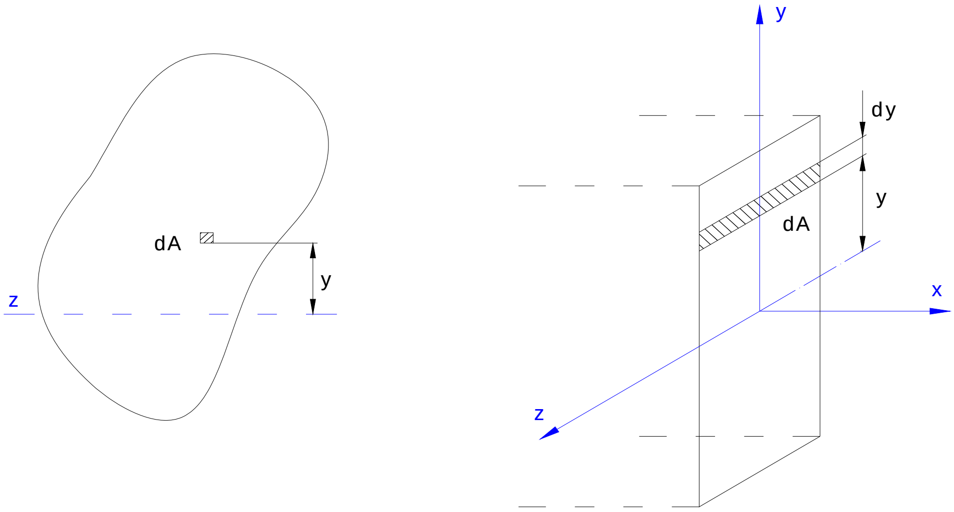


Figure 7. Calculus of the moment of inertia from axis z.

This factor will be calculated for each section of the counterweight.

4.3 Deformation by shear.

Perpendicular forces applied to a beam involve a transverse shearing stress along the beam. Consider a cantilever supporting a force *F* at a point. Figure 8 shows the deformation due to shear in a beam. At the right there is the effect of shear in a section dx wide between the support and the force, where *R* represents the reaction to *F* in the other side of the section. The shear causes a displacement of the side where the force is applied, causing a deflection deformation. This deformation is normally depreciated compared with the one caused by the bending moment, but in case of high loaded and short beams it should be taken into account.

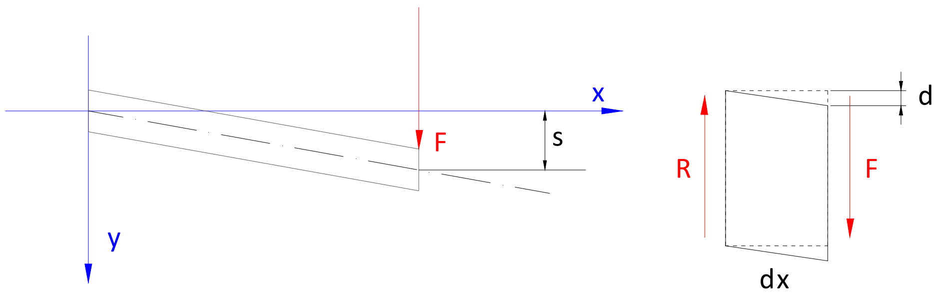


Figure 8. Shearing **d**eformation.

The deflection at a point m is:

$$s(m) = \frac{k}{A \cdot G} \cdot \int_0^m V(x) \cdot dx$$

Where *k* : shape factor of the cross-section
A : area of the cross-section
G : shear modulus of the material
V(x) : shear in the cross-section at x

V(x) is calculated as the algebraic sum of the external forces at the left side of the cross-section under study. Considering the general situation represented in Figure 6 (page 6), the shear at the right of a point x is:

$$V(x) = Q + p \cdot (l - x)$$

So the deflection at the end of the cantilever is:

$$s(l) = \frac{k}{A \cdot G} \cdot \int_0^l Q + p \cdot (l - x) \cdot dx = \frac{k}{A \cdot G} \cdot \left[Q \cdot x + p \cdot \left(l \cdot x - \frac{x^2}{2} \right) \right]_0^l$$

$$s(l) = \frac{k}{A \cdot G} \cdot \left(Q \cdot l + p \cdot \frac{l^2}{2} \right)$$

Taking into account the tilting of the cantilever as shown in Figure 3:

$$s(l) = \frac{k \cdot \cos \xi}{A \cdot G} \cdot \left(Q \cdot l + p \cdot \frac{l^2}{2} \right) \tag{6}$$

4.3.1 Shape factor.

The shape factor tries to evaluate which area of the cross-section has an effective resistance to the shear deformation. The higher the factor is, the less resistance of the cross-section and therefore the higher deformation of the beam. It depends on the shape of the cross-section. The following equation can be used for its estimation:

$$k = \frac{a \cdot A \cdot d}{I_z \cdot t}$$

Where: *a* : area beyond the neutral axis
A : area of the cross-section
d : distance of the center of gravity of the area *a* to the neutral axis.
I_z : moment of inertia of the cross-section
t : total thickness of the section

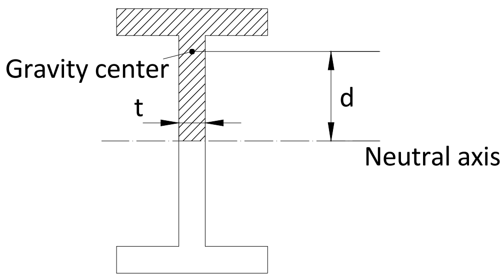


Figure 9. Shape factor data in typical I beam.

The neutral axis is that one where the longitudinal fibres of the beam keep the original length after bending. For symmetrical beams it is equivalent to the axis of symmetry. In this case *A* = 2 · *a*, so the shape factor is:

$$k = \frac{A^2 \cdot d}{2 \cdot I_z \cdot t} \tag{7}$$

5 Modeling the counterweight.

The following parameters must be determined for each section of the counterweights, in order to calculate the deformations of the structure:

- Length (*l*).
- Area (A).
- Total weight (*P*).
- Weight per unit length (*p*).
- Coefficient of linear thermal expansion (α).
- Elastic modulus (*E*).
- Shear modulus (*G*).
- Shape factor (*k*).
- Moment of inertia (I_z).

From the documents of the antenna it can be assumed that the metallic structure of the counterweight has a weight per length unit about 3.115,82 kg/m along all the cantilever.

5.1 Cantilever arm.

The parameters of the structure are taken from the antenna document and properties of the steel, except the moment of inertia that is calculated in the next paragraph. The results are the following:

$l =$ **5,840 m**

$A =$ **0,3995 m^2**

$P =$ **18.196,39 kg**

$p =$ **3.115,82 kg/m**

$\alpha =$ **1,2 · 10^{-5} ºC^{-1}**

$E =$ **200 · 10^8 kg/m^2**

$G =$ **79 · 10^8 kg/m^2**

$I_z =$ **2,8870 m^4**

$k =$ **2,0736**

5.1.1 Properties of the cantilever arm.

The following data are taken out from the documentation of the radiotelescope:

Length = 5,84 m

Weight per unit length = 3.115,82 kg/m

So the weight is = 3.115,82 · 5,84 = 18.196,39 kg

Because the structure is completely made of steel, it takes the following properties from that material:

E = 200 kg/m^2

Density = 7.800 kg/m^3

α = 1,2 · 10^{-5} ºC^{-1}

The area of the cross-section can be calculated from the total weight, the length and the density of the steel;

$$A = \frac{volume}{length} = \frac{mass}{density \cdot length} = \frac{18.196{,}39}{7.800 \cdot 5{,}84} = 0{,}3995\ m^2$$

5.1.2 Moment of inertia of the cantilever arm.

The moment of inertia depends heavily on the shape of the cross-section of the structure. The structure has several plates and beams diagonal and perpendicular to the cantilever axis, some of them with holes to allow access to the interior. Such complex structure makes the most of the cross-sections be unique along the counterweight. In order to simplify the calculations, the same cross-section must be supposed along all the structure. Four conditions have been taking into account for determining the simplified cross-section:

- The area of material must be according to the value previously calculated.
- The cross-section is symmetrical.
- There are six plates along the structure parallel to the longitudinal and to z axes of the counterweight.
- The thickness of the external walls is 15 mm, the real thickness increased due to the L beams strengthening the surface of the walls.

Then, the thickness of each horizontal plate is:

$$e_1 = \left(\frac{A}{2} - A_A - 2 \cdot h_2 \cdot e_2\right) \cdot \frac{1}{3} \cdot \frac{1}{w - 2 \cdot e_2} = 10{,}9\ mm$$

Where A_A is the area of the section A (see Figure 11):

$$A_A = \frac{w.h}{2} - \frac{w'.h'}{2} = 0{,}0359\ m^2$$

The resultant cross-section is shown in Figure 10. It can be divided in simplex shapes which moment it easy to calculate.

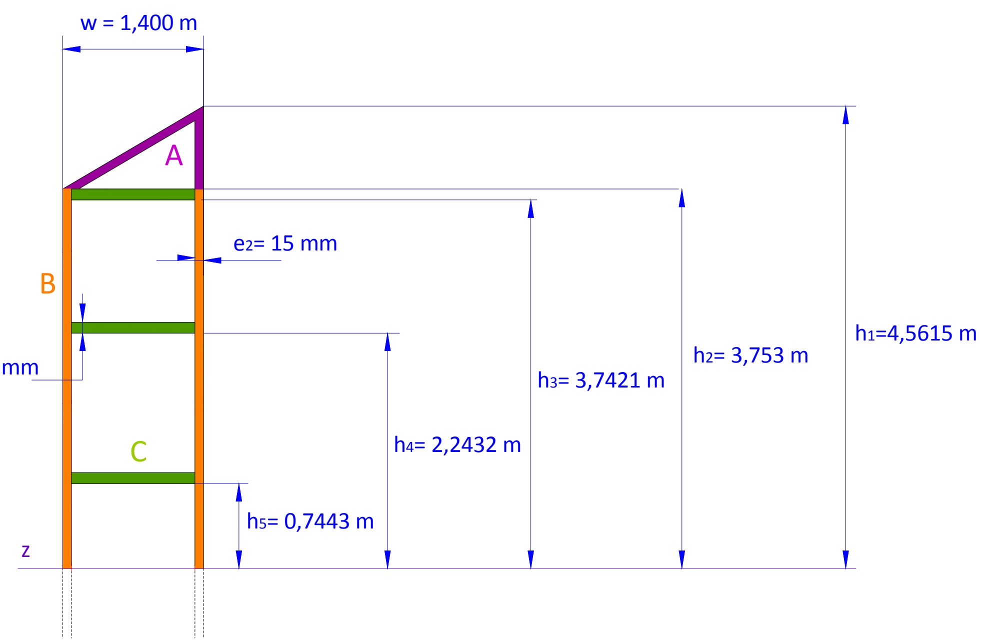
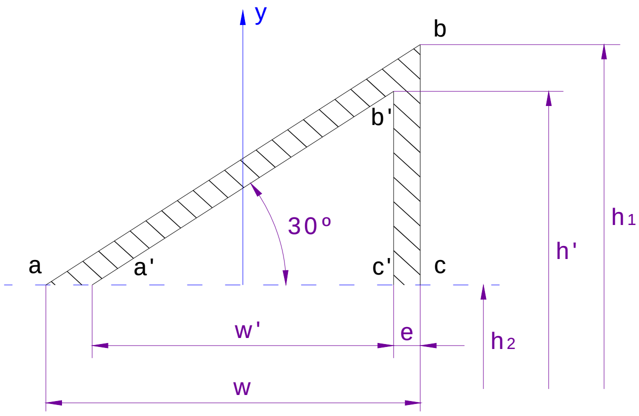


Figure 10. Upper half of the cross-section of the cantilever arm.

The total moment will be the sum of all partial moments. Lets divide the shape in a V shape A, two rectangles B and three rectangles C at different distances from axis z as shown in the figure.

The moment of inertia of A can be calculated as the moment of a solid triangle abc minus the moment of a triangle a'b'c' as shown in the Figure 11.



$h_1 = 4{,}5615$ m

$w = 1{,}400$ m

$e = 0{,}015$ m

$w' = w - e - e/\sin 30 = 1{,}3550$ m

$h' = h_2 + w' \cdot \text{tg } 30 = 4{,}5353$ m

Figure 11. Triangular shape of the cross section.

The width of any infinitesimal section parallel to the base and thickness *dy* for triangle abc is:

$$width = \frac{h_1 - y}{h_1 - h_2} \cdot w$$

So the moment of inertia of the triangle from axis z is:

$$I_{z')abc} = \int_{h_2}^{h_1} y^2 \cdot \left(\frac{h_1 - y}{h_1 - h_2} \cdot w\right) \cdot dy = \frac{w}{h_1 - h_2} \left[\frac{h_1 \cdot y^3}{3} - \frac{y^4}{4}\right]_{h_2}^{h_1} =$$

$$= \frac{w}{h_1 - h_2} \cdot \left[\left(\frac{{h_1}^4}{3} - \frac{{h_1}^4}{4}\right) - \left(\frac{h_1 \cdot {h_2}^3}{3} - \frac{{h_2}^4}{4}\right)\right] = \frac{w \cdot \left({h_1}^4 - 4 \cdot h_1 \cdot {h_2}^3 + 3 \cdot {h_2}^4\right)}{12 \cdot (h_1 - h_2)} = 9{,}1779\, m^4$$

In the same way, the moment of the triangle a'b'c' from axis z is:

$$I_{z')a'b'c'} = \frac{w' \cdot \left(h'^4 - 4 \cdot h' \cdot {h_2}^3 + 3 \cdot {h_2}^4\right)}{12 \cdot (h' - h_2)} = 8{,}5566\, m^4$$

So, the moment of inertia of the shape A from axis z is:

$$\boldsymbol{I_{z'A}} = I_{z)abc} - I_{z)a'b'c'} = \mathbf{0,6213\, m^4}$$

The moment of inertia of a rectangle *w* wide and with lower side h_L and upper side h_U from de axis is:

$$I_z = \int_{h_L}^{h_U} y^2 \cdot w \cdot dy = \frac{w \cdot \left({h_U}^3 - {h_L}^3\right)}{3}$$

So the moment of inertia of one rectangles B is:

$$\boldsymbol{I_{zB}} = \frac{e_2 \cdot {h_2}^3}{3} = \mathbf{0,2643\, m^4}$$

And the moment of inertia of each rectangles C are:

$$\boldsymbol{I_{zC1}} = \frac{(w - 2 \cdot e_2) \cdot \left({h_2}^3 - {h_3}^3\right)}{3} = \mathbf{0{,}2097}\ \boldsymbol{m^4}$$

$$\boldsymbol{I_{zC2}} = \frac{(w - 2 \cdot e_2) \cdot \left((h_4 + e)^3 - {h_4}^3\right)}{3} = \mathbf{0{,}0755}\ \boldsymbol{m^4}$$

$$\boldsymbol{I_{zC3}} = \frac{(w - 2 \cdot e_2) \cdot \left((h_5 + e)^3 - {h_5}^3\right)}{3} = \mathbf{0{,}0084}\boldsymbol{m^4}$$

Taking into account that the above calculations were for the upper middle of the cross-section, the total moment of inertia of the cross-section is:

$$\boldsymbol{I_z} = 2 \cdot (I_{zA} + 2 \cdot I_{zB} + I_{zC1} + I_{zC2} + I_{zC3}) = \mathbf{2{,}8870}\ \boldsymbol{m^4}$$

5.1.3 Shape factor of the cantilever arm.

The height of the center of gravity of Figure 10 can be calculated as that horizontal line that has the same effective area above and below. This corresponds to a height of 2,2535 m from axis z.

The total thickness of the cross-section is twice the thickness of the external walls, that is, 0.03 m.

So the shape factor of the cantilever arm is (see equation (7)):

$$\boldsymbol{k} = \frac{0.3995^2 \cdot 2.2535}{2 \cdot 2{,}8870 \cdot 0.03} = \mathbf{2{,}0763}$$

5.2 Ballast carrier.

The dimensions and weights of the ballast carrier are obtained from the technical documents of the radiotelescope. The physical coefficients are calculated taking into account the mixture of materials of the ballast. All parameters are calculated in the following paragraphs. They are:

l = 3,1 m

A = 8,6163 m^2

P = 104.759 kg

p = 33.793 kg/m

α = $1{,}2 \cdot 10^{-5}$ $^{o}C^{-1}$

E = $64{,}0493 \cdot 10^8$ kg/m^2

G = $17{,}336 \cdot 10^8$ kg/m^2

I_z = 50,5234 m^4

k = 1,4918

5.2.1 Properties of the ballast carrier.

From the documentation of the radiotelescope:

The required weight of the ballast for a correct compensation is 95.100 kg.

The length of the ballast carrier is 3,1 m

The weight of the structure per unit length 3.115,82 kg/m

So the weight of the structure is W_{str} = 3.115,82 · 3,1 = 9.659,04 kg

Taking into account the density of steel, the volume of the structure is V_{str} = 9.656,04 / 7.800 = 1,238 m^3

The weight of the ballast carrier is P = 95.100 + 9.659,04 = 104.759 kg

And the weight per unit length is p = 104.759 / 3,1 = 33.793 kg/m

The area of the cross-section can be calculated from Figure 12 as the area of the rectangle B plus the area of the triangle A minus the grey area;

$$A = 2 \cdot \left(w \cdot h_3 + \frac{w \cdot (h_1 - h_3)}{2} - e \cdot h_3 - e \cdot h_2 \right) = 8{,}6163\ m^2$$

So the volume of the ballast carrier is $V_B = A \cdot l$ = 26,7105 m^3

The ballast is a mix of small pieces of steel and concrete. Its properties are estimated according to the rule of mixtures. This rule predicts the properties as the volume weighted average of the components properties. It is assumed that there is not any air gap into the mixture and that the mix is homogeneous in all the ballast.

The properties of each component are:

	Steel	Concrete
Modulus of elasticity	E_S =200·10^8 kg/m^2	E_C = 30·10^8 kg/m^2
Shear modulus	G_S =79·10^8 kg/m^2	G_C = 12,5·10^8 kg/m^2
Density	ρ_S = 7.800 kg/m^3	ρ_C = 2.000 kg/m^3
Coefficient of thermal expansion	α_S = 1,2 · 10^{-5} $^oC^{-1}$	α_C = 1,2 · 10^{-5} $^oC^{-1}$

The first step is to calculate the proportion of the volume for each component in the mixture. For this calculation, the steel of the structure is also taken into account.

The volume of each component in the ballast carrier is:

$$\text{Steel}: \ \rho_S \cdot V_S + \ \rho_C \cdot (V_B - V_S) = P \ \Rightarrow \ V_S = 8{,}8514\ m^3 \ \Rightarrow \ v_S = \frac{V_S}{V_B} = 0{,}3314$$

$$\text{Concrete}: \ v_C = 1 - \ v_S = 0{,}6686$$

According to the rule of mixtures, the properties are the following:

- The modulus of elasticity depends on the orientation of the steel particles with respect the counterweight axis. For longitudinal orientation (subindex L) the coefficient adopts its maximum value, and for transverse orientation (subindex T) it takes the minimum value. A random orientation of the particles in the mixture can be assumed. For this reason, the average of both values is adopted as the most approximate value:

$$E_L = \ E_S \cdot v_S + E_C \cdot v_C = 86{,}3348 \cdot 10^8 \ \text{kg/m}^2$$

$$E_T = \frac{E_S \cdot E_C}{E_S \cdot v_C + E_C \cdot v_S} = 41{,}7638 \cdot 10^8 \ \text{kg/m}^2$$

$$\boldsymbol{E} = \frac{E_L + E_T}{2} = \mathbf{64{,}0493 \cdot 10^8 \ kg/m^2}$$

- The shear modulus is:

$$\boldsymbol{G} = \frac{G_S \cdot G_C}{G_S \cdot v_C + G_C \cdot v_S} = \mathbf{17,3360 \cdot 10^8\ kg/m^2}$$

- The linear thermal coefficient is taken the same for both materials, so the coefficient of the mixture will be that value.

5.2.2 Moment of inertia of the ballast carrier.

Figure 12 shows the simplified shape of the cross-section of the ballast carrier. Dimensions have been taken from the radiotelescope documentation. It can be divided in simplex forms which moment easy to calculate. The total moment will be the sum of all partial moments. Lets divide the shape in a triangle shape A, two rectangles B and three rectangles C at different distances from axis z. The moment is the sum of the moments of both shapes minus the moment of the empty area denotes grey in the figure.

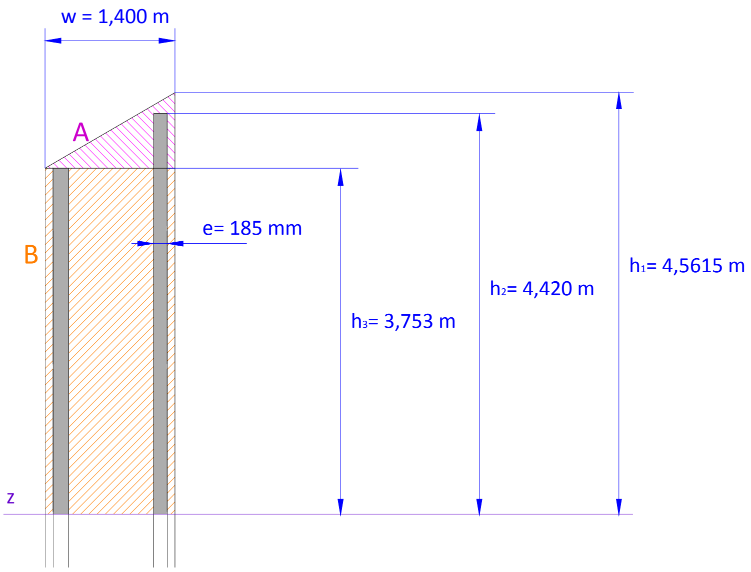


Figure 12. Upper half of the cross-section of the ballast carrier.

The moment of inertia of the triangle A from axis z is:

$$\boldsymbol{I_{zA}} = \frac{w \cdot \left(h_1{}^4 - 4 \cdot h_1 \cdot h_3{}^3 + 3 \cdot h_3{}^4\right)}{12 \cdot (h_1 - h_3)} - \frac{e \cdot \left(h_2{}^3 - h_3{}^3\right)}{3} = \boldsymbol{7,1127\ m^4}$$

The moment of inertia of shape B is:

$$\boldsymbol{I_{zB}} = \frac{(w - 2 \cdot e) \cdot h_3{}^3}{3} = \boldsymbol{18,149\ m^4}$$

Taking into account that the above calculations were for the upper middle of the cross-section, the total moment of inertia of the whole cross-section is:

$$\boldsymbol{I_z} = 2 \cdot (I_{zA} + I_{zB}) = \boldsymbol{50,5234\ m^4}$$

5.2.3 Shape factor of the ballast carrier.

The height of the center of gravity of Figure 12 can be calculated as that with the same effective area above and below. This corresponds to a height of 2,0915 m from axis z.

The total thickness of the cross-section is:

$$t = w - 2 \cdot e = 1{,}03\ m$$

So the shape factor of the cantilever arm is (see equation (7)):

$$\boldsymbol{k} = \frac{8{,}617^2 \cdot 2.0915}{2 \cdot 50{,}5234 \cdot 1.03} = \mathbf{1{,}4918}$$

6 Evaluation of the deformation.

Consider the position of the counterweight defined by the vector $\vec{v}$. Consider a longitudinal deformation δ and a transversal deformation f of the counterweight as shown if Figure 13. Then the deformation of the counterweight is represented by the vector $\vec{\varepsilon}$.

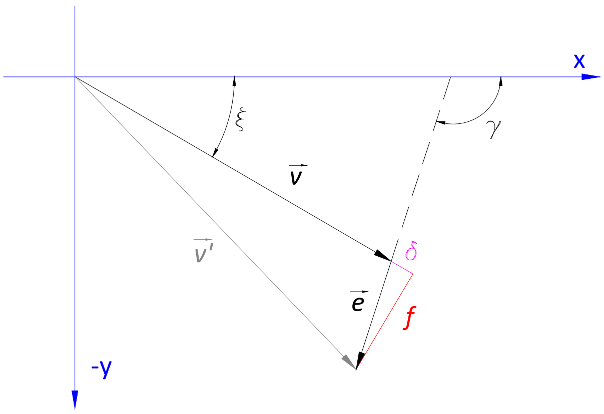


Figure 13. Deformation of the counterweight.

6.1 Deformation by ambient temperature change.

In paragraph 3 (page 2) the deformation by ambient temperature changes was defined as:

$$\Delta l = \alpha_T \cdot l \cdot \Delta T$$

The deformation by stress appears more quickly that the one by change of ambient temperature. Therefore, in the calculation of the elongation by change of ambient temperature, the initial length of the structure should be the one after the elongation by tension. However, the elongation by tension so small to introduce changes in the elongation by ambient temperature changes, so the original length will be used in the formula.

1. For the cantilever arm the parameters are:
   - L : length of the structure = 5,84 m
   - α_T : coefficient of linear thermal expansion = 1,2 · 10^{-5} $^oC^{-1}$

Replacing the values in the equation:

$$\Delta \boldsymbol{l_{CA}} = \mathbf{0{,}0701} \cdot \Delta \boldsymbol{T} \ [\boldsymbol{mm}]$$

2. For the ballast carrier the parameters are:
   - L : length of the structure = 3,1 m
   - α_T : coefficient of linear thermal expansion = 1,2 · 10^{-5} $^oC^{-1}$
   - ΔT : change of ambient temperature = 25 ^oC.

Replacing the values in the equation:

$$\Delta \boldsymbol{l_{BC}} = \mathbf{0{,}0372} \cdot \Delta \boldsymbol{T} \ [\boldsymbol{mm}]$$

6.2 Deformation by tension.

In paragraph 4.1 (page 4) the deformation by tension was defined as:

$$\delta = \frac{l}{A \cdot E} \cdot \left(\frac{P \cdot \sin\xi}{2} + Q \cdot \sin\xi\right)$$

1. For the cantilever arm the parameters are:
   - l : length of the structure = 5,84 m
   - A : Area of the cross-section = 0,3995 m^2
   - E : Elastic coefficient = $200 \cdot 10^8$ kg/m
   - P : Weight of the structure = 18.196,39 kg
   - Q : Weight of the ballast box = 104.759 kg
   - ξ : tilting angle = elevation angle of the antenna.

   Replacing the values in the equation:

$$\boldsymbol{\delta_{CA} = 0,0833 \cdot \sin\xi\ [mm]}$$

2. For the ballast carrier the parameters are:
   - l : length of the structure = 3,100 m
   - A : Area of the cross-section = 8,6163. m2
   - E : Elastic coefficient = 64,0493 $\cdot 10^8$ kg/m
   - P : Weight of the structure = 104.759 kg
   - Q = 0 kg
   - ξ : tilting angle = elevation angle of the antenna.

   Replacing the values in the equation:

$$\boldsymbol{\delta_{BC} = 0,0029 \cdot \sin\xi\ [mm]}$$

6.3 Deformation by bending.

In paragraph 4.2 (page 5) the deformation by bending was defined by an angle θ and a deflection *f*. given by the equations:

$$\theta(l) = \frac{1}{E \cdot I_z} \cdot \left(M \cdot l + Q \cdot \frac{l^2}{2} \cdot \cos\xi + p \cdot \frac{l^3}{6} \cdot \cos\xi\right) \cdot \frac{180}{\pi} \quad [deg]$$

$$f(l) = \frac{1}{E \cdot I_z} \cdot \left(M \cdot \frac{l^2}{2} + Q \cdot \frac{l^3}{3} \cdot \cos\xi + p \cdot \frac{l^4}{8} \cdot \cos\xi\right)$$

1. For the cantilever arm the parameters are:
   - l : length of the structure = 5,840 m
   - E : Elastic coefficient = $200 \cdot 10^8$ kg/m^2
   - I_z : Moment of inertia = 2,8870 m^4
   - p : Weight per unit length = 3.115,82 kg/m
   - M : Moment at the end of the structure.
   - Q : Vertical force at the end of the structure
   - ξ : tilting angle = elevation angle of the antenna.

The moment M in the connection structure is generated by the weight of the ballast carrier. This moment is the following:

$$M = \int_0^{l_{BC}} x \cdot (q_{BC} \cdot \cos\xi \cdot dx) = q_{BC} \cdot \cos\xi \cdot \frac{{l_{BC}}^2}{2} = 33.793 \cdot \cos\xi \cdot \frac{3.1^2}{2} = 162.375 \cdot \cos\xi \quad [\mathrm{kg \cdot m}]$$

Q is the weight of the ballast box = 104.759 kg

Replacing in the equations:

$$\boldsymbol{\theta_{CA} = 0,0027 \cdot \cos\xi \quad [\mathrm{deg}]}$$

$$\boldsymbol{f_{CA} = 0,1763 \cdot \cos\xi \quad [mm]}$$

2. For the ballast carrier the parameters are:
   - l : length of the structure = 3,100 m
   - E : Elastic coefficient = 64,0493·10^8 kg/m^2
   - I_z : Moment of inertia = 50,5234 m^4
   - p : Weight per unit length = 33.793 kg/m
   - M : Moment at the free end = 0.
   - Q : Force at the free end = 0
   - ξ : tilting angle = elevation angle of the antenna.

Replacing in the equations:

$$\boldsymbol{\theta_{BC} = 0,00003 \cdot \cos\xi \quad [\mathrm{deg}]}$$

$$\boldsymbol{f_{BC} = 0,0012 \cdot \cos\xi \quad [mm]}$$

6.4 Deformation by shear.

In paragraph 4.2 (page 5) the deformation due to bending was defined by a deflection y as:

$$s(l) = \frac{k \cdot \cos\xi}{A \cdot G} \cdot \left(Q \cdot l + p \cdot \frac{l^2}{2}\right)$$

1. For the cantilever arm the parameters are:
   - l : length of the structure = 5,840 m
   - A : Area of cross-section = 0,3995 m^2
   - G : Shearing coefficient = 79·10^8 kg/m^2
   - k : Shape factor = 2,0736
   - p : Weight per unit length = 3.115,82 kg/m
   - Q : Weight of the ballast box = 104.759 kg
   - ξ : tilting angle = elevation angle of the antenna.

   Replacing in the equation:

$$\boldsymbol{s_{CA} = 0,4374 \cdot \cos\xi \quad [mm]}$$

2. For the ballast carrier the parameters are:
   - l : length of the structure = 3,1 m
   - A : area = 8,6163 m^2
   - G : Shearing coefficient = 17,336·10^8 kg/m^2
   - k : Shape factor = 1,4918
   - p : Weight per unit length = 33.793 kg/m
   - Q : Perpendicular force at the free end = 0
   - ξ : tilting angle = elevation angle of the antenna.

   Replacing in the equation:

$$\boldsymbol{s_{BC}} = \mathbf{0,0162 \cdot \cos\xi} \; [\boldsymbol{mm}]$$

6.5 Total deformation.

The following table resume the contribution in millimeters of each effect in each section:

Effect	Cantilever arm	Ballast carrier
Elongation by ΔT_{amb}	$0{,}0701 \cdot \Delta T$	$0{,}0372 \cdot \Delta T$
Elongation by tension	$0{,}0833 \cdot \sin\xi$	$0{,}0029 \cdot \sin\xi$
Deflection by bending	$0{,}1763 \cdot \cos\xi$	$0{,}0012 \cdot \cos\xi$
Deflection by shear	$0{,}4374 \cdot \cos\xi$	$0{,}0162 \cdot \cos\xi$

The total deflection of each section is the sum of its deflection by bending and by shear:

$$f_T)_{CA} = (0{,}1763 + 0{,}4374) \cdot \cos\xi \; = 0{,}6137 \cdot \cos\xi \; [mm]$$

$$f_T)_{BC} = (0{,}0012 + 0{,}0162) \cdot \cos\xi \; = 0{,}0174 \; \cdot \cos\xi \; [mm]$$

The deflection of the cantilever arm supposes an additional tilting of the ballast carrier. Assuming this angle very small, we can calculate the consequent deflection at the end of the counterweight as a linear projection of the deflection of the cantilever arm. The total deflection of the counterweight is then:

$$\boldsymbol{f} = f_T\big)_{CA} \cdot \frac{l_{CA} + l_{BC}}{l_{CA}} + f_T)_{BC} = \mathbf{0,9569\cos\xi} \;\; [\boldsymbol{mm}]$$

The total elongation of the counterweight is the sum of the elongations by tension plus the elongations by temperature:

$$\delta = \delta_{CA} + \delta_{BC} + \delta_T$$

$$\boldsymbol{\delta} = \mathbf{0,0862 \cdot \sin\xi + 0,1073 \cdot \Delta}\boldsymbol{T_{amb}} \;\; [\boldsymbol{mm}]$$

The following Figure shows the relationship between both deformations and the elevation of the antenna for an ambient temperature of 25 ^{o}C, that is, ΔT_{amb} = 0 ^{o}C.

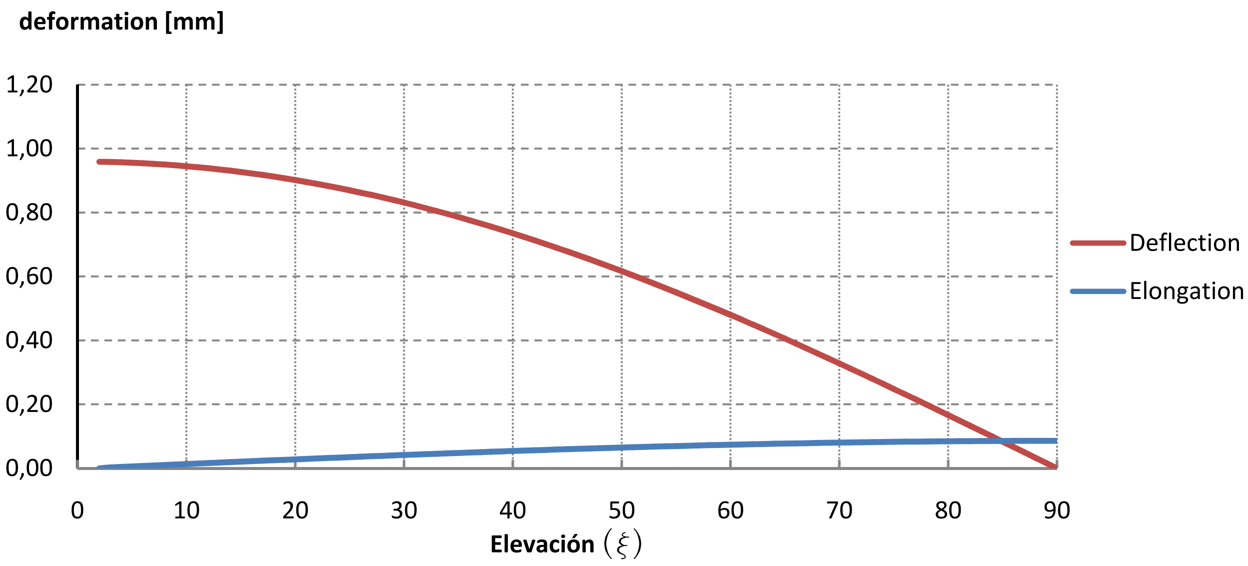
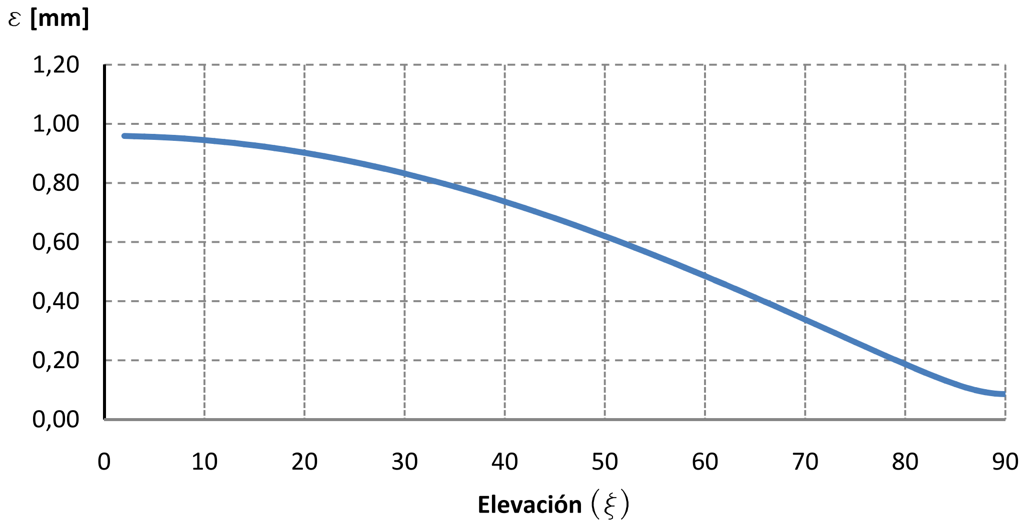


Figure 14. Total deflection and elongation with respect elevation.

The modulus and angle respect the coordinate system of the deformation vector of the counterweight are shown in Figure 15. The graphics at the right shows the modulus and angle of the deformation vector for a null change of ambient temperature.

$$\varepsilon = \sqrt{\delta^2 + f^2}$$



$$\gamma = \xi + \tan^{-1}\frac{f}{\delta}$$

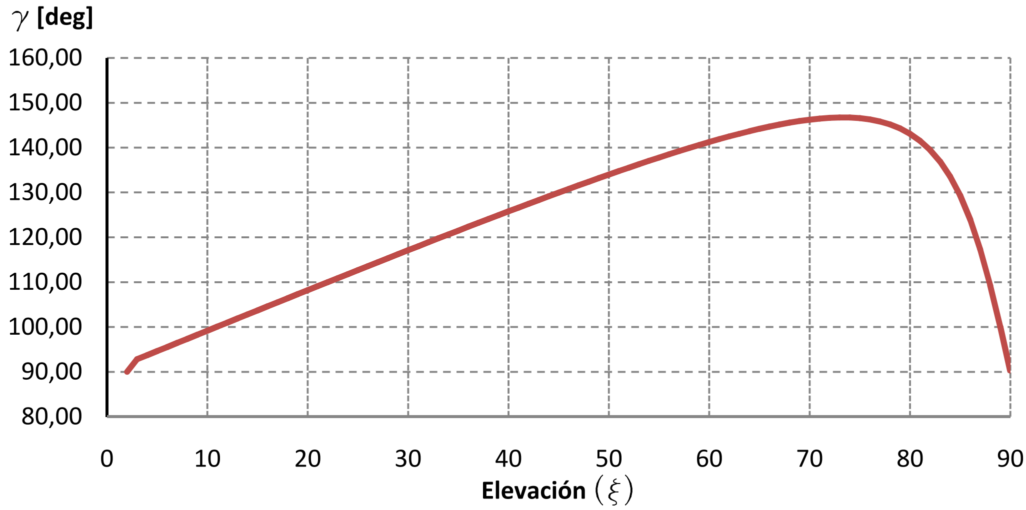


Figure 15. Deformation at the free end of the counterweight.

7 Conclusions.

The change of ambient temperature in the short time at the Observatorio de Yebes is between 6 ^{o}C and 21,5 ^{o}C in a day. In the structure some measurements have given 10 ^{o}C more than ambient temperature due to sun radiation. Therefore, the temperature in the structure can changes between 6 ^{o}C and 31,5 ^{o}C, what means a change in the length of the counterweight between 0,64 and 3,38 mm. In winter, the effect of temperature along a day is similar to the effect of gravity. In summer, it can be three times greater.

The gravity deforms the counterweight depending on the elevation of the antenna. The most of the contribution to the deformation comes from the cantilever arm. In fact, the deformation of the cantilever arm supposes about the 96 % of the total elongation and about the 97 % of the total deflection.

The deformation by shear is greater than the deformation by bending. This is normal in structures which length is comparable to its height. In fact, the deflection by shear in the cantilever arm is more than twice the one by bending and in the ballast container about thirteen times.

The deflection is most significant than elongation taking into account only gravitational deformation. It is higher than elongation when the elevation is below than 85 degrees, and more than twice below 79 degrees.

The estimation of the cross-section shape influences in the values of the moment of inertia and shear factor, and therefore in the results. The estimation is especially complicated for the cantilever arm, where the structure is more complex. Other estimations can be done. The most relevant is that which is clearly less rigid than real one, so it gives an idea of a maximum value for expected deformation. This estimation can be the one on Figure 10 without the green rectangles C and with the real thickness for walls, which is 10mm. For this shape, the values are:

- Area = 0.2 m^2
- Moment of inertia = 1,6 m^4
- Shape factor = 1,44
- Elongation by gravity = 0,168 · sin ξ mm
- Deflection by shear = 0,61 · cos ξ mm
- Deflection by bending = 0,33 · cos ξ mm
- Total elongation = 0,171 · sin ξ mm
- Total deflection = 1,46 · cos ξ mm

8 References.

[REF-1] Santamaría-Gómez, A., and S. García-Espada (2011). *Simulating the estimation of the 40m radio-telescope Invariant Reference Point at the Yebes observatory*.

IT-OAN 2011-9.

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Annex I. Drawings.

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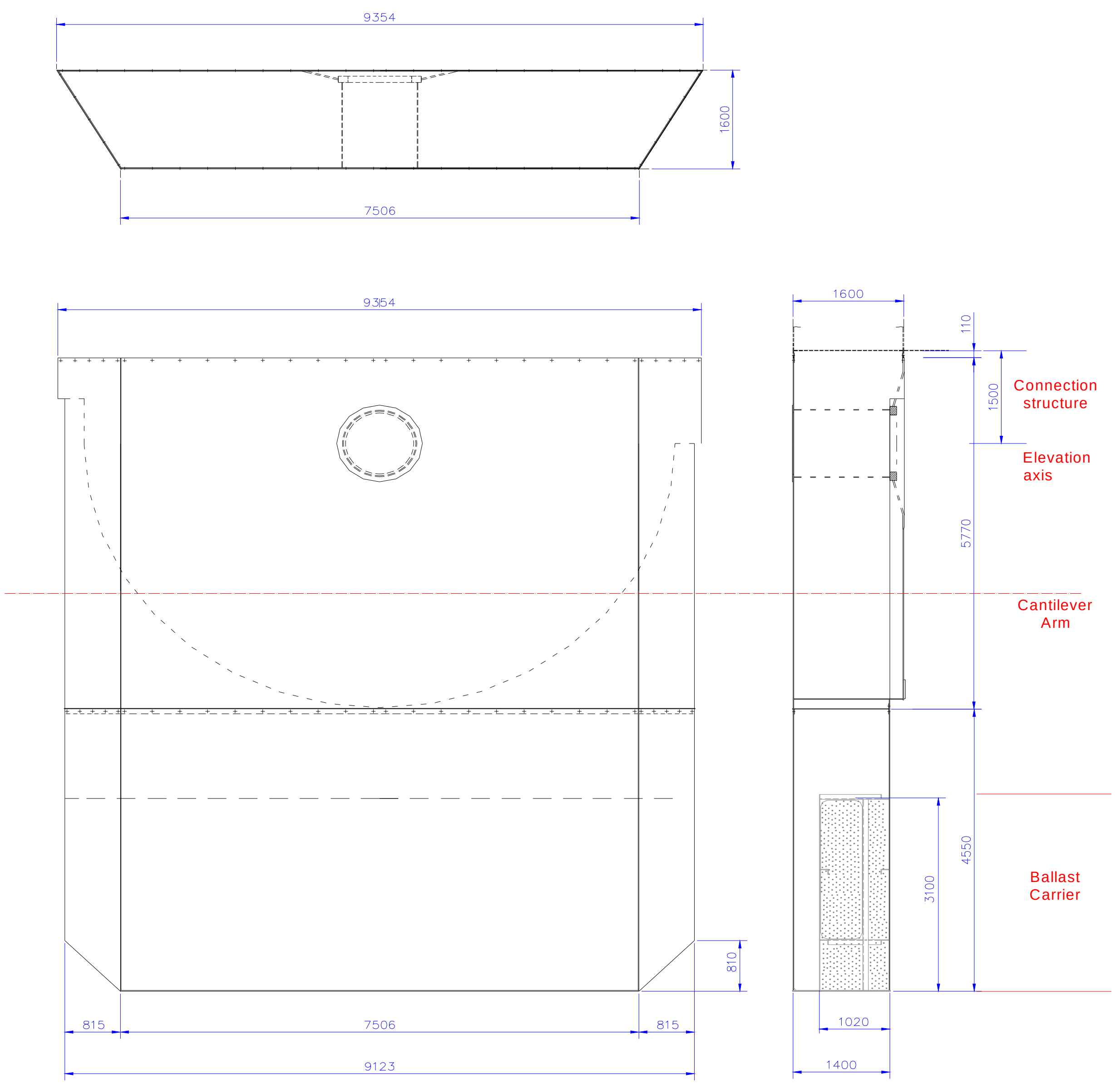

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CENTRO DE DESARROLLOS TECNOLÓGICOS

PROYECTO: **Vertical deformation of the cantilever in the 40m radiotelescope.**

REALIZADO POR: *Carlos Albo*		DESCRIPCIÓN: Dimensions of the cantilever	PLANO: IT-OAN-2012-9-001
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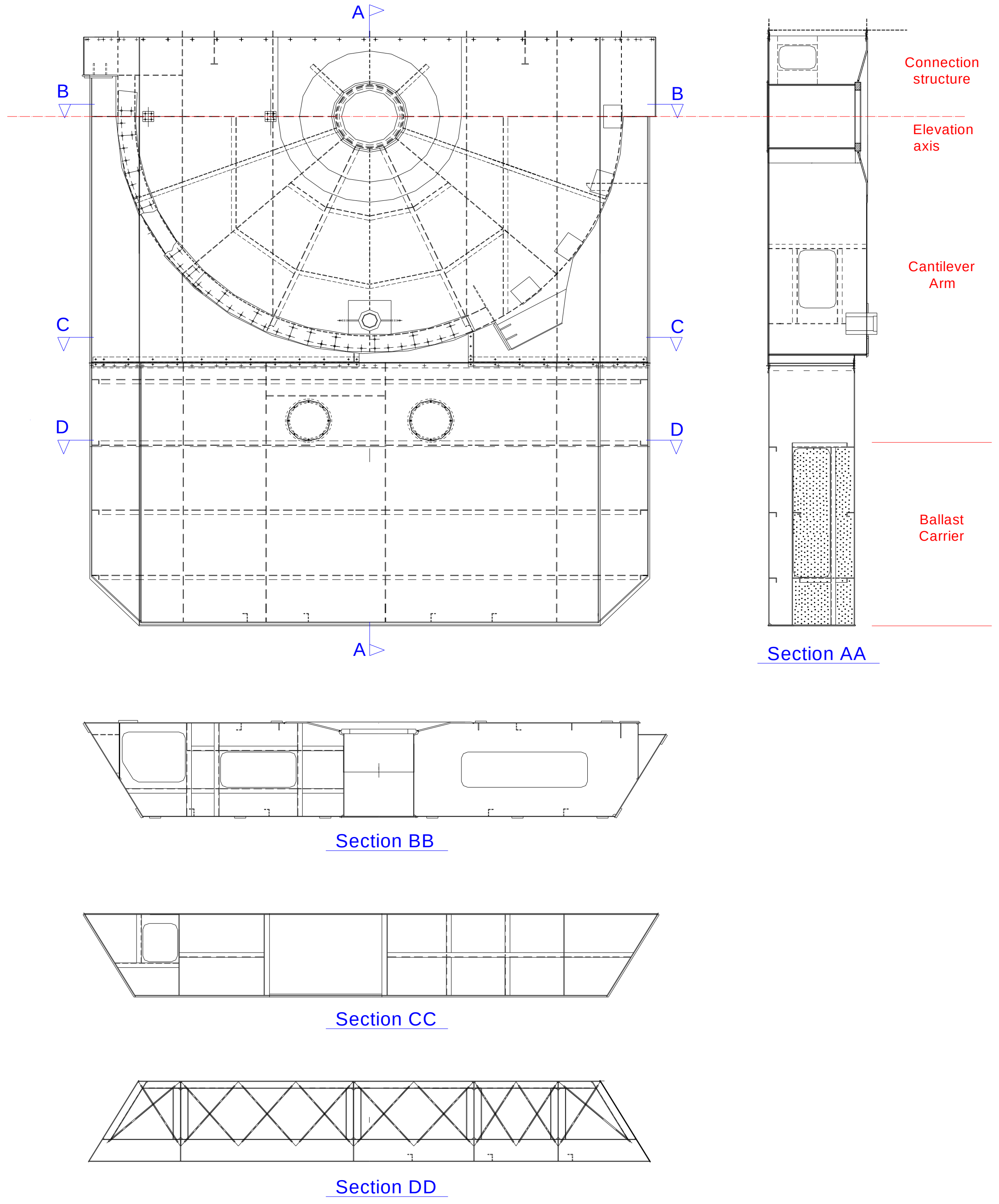

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