Simulating the estimation of the 40m radio-telescope Invariant Reference Point at the Yebes observatory

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Revision history

Version	Date	Author	Updates
1.0	24-08-2011	ASG & SGE	First version
2.0	03-10-2011	ASG & SGE	Second version : Corrected bug on the random errors applied to the simulated field observations.

Contents

1 Introduction

The position of the instrumental reference point is a key issue in the determination of the local tie(s) connecting all the observing geodetic techniques at Yebes observatory. This document describes the simulations carried out to validate the methodology and the observing strategy in order to estimate the location, in a local coordinate system, of the invariant reference point of the 40m radio-telescope at Yebes observatory. However, the outcome of these simulations can be applied to any other radio-telescope elsewhere.

In Section 2 we describe why is so important to determine the local tie between co-located geodetic instruments and which is the geodetic reference point of a radio-telescope. In Section 3 we describe the current situation at Yebes observatory. In Section 4 we describe the geometric model used for the estimation of the invariant reference point of a radio-telescope. In Section 5 we describe the procedure to simulate the field observations under different observing scenarios. Finally, in Section 6 we present the results of the simulation and the conclusions about the best approach to estimate the invariant reference point of a radio-telescope.

2 What is a local tie? (and why should we care?)

Most of the space-based and ground-based Earth observations, such as precisely determining satellite orbits, monitoring Earth rotation, modeling tectonic plate motion or assessing sea level rise and its variability in space and time, fundamentally depend on the availability of a global Terrestrial Reference System (TRS) that only space geodesy is able to realize [*Altamimi et al.*, 2011]. A TRS is defined as an affine trihedron close to the Earth and co-rotating with it in its diurnal motion in space within the framework of the General Relativity [*IERS Conventions*, 2010]. In such a system, the positions of points attached to the solid surface of the Earth undergo only small variations with time, due mainly to geophysical effects (e.g. tectonic or loading). In the other hand, the definition of such trihedron through the materialization of its origin (e.g. the Earth's center of mass or geocenter), orientation axes, scale and their time evolution is known as a Terrestrial Reference Frame (TRF) or, equivalently in geodesy, a datum. This means that while a TRS denotes a conventional error-free theoretical mathematical definition, the TRF is achieved by a set of coordinate and velocity values assigned to given physical points on the Earth's surface which are observed using space geodesy techniques (hence with errors and uncertainties). The coordinates of these points together with their linear variations over time (i.e. their velocities) materialize the position of the TRS in space. From these coordinates and velocities, any other terrestrial frame can be defined and related to the TRF through a similarity transformation of 14 parameters, namely: three translations and their rates for the origin, three rotations and their rates for the orientation of the axes, the scale and the scale rate.

These physical points on the Earth's surface correspond with the reference points of geodetic stations of different techniques. The four space geodetic techniques currently used to realize a geometric TRF are: Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS), Satellite Laser Ranging (SLR), Global Navigation Satellite Systems (GNSS), and

Very Long Baseline Interferometry (VLBI). Spatial gravimetry and spatial altimetry are also space geodetic techniques, but they are not taken into account to realize a geometric TRS.

The network of each of these four techniques defines its own TRF computed by its corresponding Technique Service within the International Association of Geodesy (IAG): the International DORIS Service (IDS), the International Laser Ranging Service (ILRS), the International GNSS Service (IGS), and the International VLBI Service (IVS). However none of these techniques is capable of fully realize a TRS because none of them is capable of define by itself all the parameters needed. In addition, some of these parameters are better defined by some techniques than others, but as a whole, none of them is superior to the others. For instance, although all the dynamical techniques (SLR, GNSS and DORIS) have theoretically access to the Earth's center of mass (point around which orbit the observed satellites), only SLR is capable of providing nowadays a precise estimation of the origin of the TRS. In the same way, VLBI and SLR are the most adequate techniques to estimate the scale parameter, being an externally fixed parameter for GNSS. Finally, no reference system (terrestrial or celestial) has an intrinsic defined orientation [*IERS Conventions*, 2010] and thus, it must be fixed by convention through the International Astronomical Union resolutions. The most adequate techniques to relate the instantaneous orientation of the Earth to the conventionally fixed orientation of the TRS are VLBI (for UT1-UTC, precession and nutation) and GNSS (for polar motion or wobble).

The determination of the parameters defining a TRS is of capital importance because any bias or drift in any of them will propagate into misleading geophysical results as, for instance, the origin and the scale parameters to assess the sea level change. However, as described above, none of the four space geodetic techniques by itself can provide an accurate and reliable TRF. This goal is only possible using the advantages of all of them together through a multi-technique combined realization of the TRF, such as the International Terrestrial Reference Frame (ITRF). Only the combination of the TRF of each technique allows the best ITRF realization possible. In this sense, the International Terrestrial Reference System (ITRS), realized by successive ITRF combinations, has been formally adopted and recommended for all Earth science applications (Resolution number 2 of Perugia IUGG General Assembly 2007).

The ITRF is regularly updated by the International Earth Rotation and Reference Systems Service (IERS) to take into account not only new accumulated data, but also improved data analysis strategies of each technique. The last realization released by the IERS in 2010 has been the ITRF2008 [*Altamimi et al.*, 2011] covering a data span from the beginning of space geodesy ($\sim$1980) to 2008. This work ultimately falls under the auspices of the IERS thanks to the solutions provided by the technique services. However, these services are international voluntary organizations of scientists and other professionals. It is therefore incumbent upon national geodetic agencies to perform a variety of tasks within the IERS so that the ITRF, as the foundation of their national datums, is robust and capable of serving the ever-greater accuracy needs of the geospatial community. In this sense, many countries have officially adopted geodetic reference systems that rely upon some realization of the ITRS. For example, Spain [Real Decreto 1071/2007, de 27 de julio] and most of the European countries have adopted the realization of the European Terrestrial Reference System of 1989 (ETRS89) which was defined in terms of a 14-parameter similarity transformation with respect to the ITRF at epoch 1989.0.

The most frequent method for a multi-technique combination is through co-locating several instruments of different techniques at a subset of sites on Earth. It is worth noting that in this sense the GNSS technique plays a major role in the ITRF combination. Certainly, for the ITRF2008 realization there was only 8 co-location sites between SLR and VLBI networks, however, there was up to 48 co-locations between GNSS and SLR and 44 between GNSS and VLBI networks [*Altamimi et al.*, 2011]. This way, only the GNSS technique allows for a proper combination of the other three space geodetic techniques. Unfortunately, co-location by itself is not enough for the combination. The actual geometric 3-dimensional local vector tying the physical reference points of each instrument must be accurately known. This 3-dimensional vector in known as the local tie. In the ITRF combination, the available local ties are used as additional observations with their proper variances together with the TRF solutions provided by each technique.

The determination of the local ties is one of the above-mentioned tasks of the national geodetic agencies collaborating with the IERS. This determination is usually carried out by highly precise local terrestrial surveys. This way, many different groups have performed terrestrial surveys campaigns to determine the local tie between the physical reference points of different geodetic techniques, generally on sites of national or international interest [e.g. *Johnston et al.*, 2000; *Sarti et al.*, 2004; *Haas and Eschelbach*, 2005; *Shibuya et al.*, 2005; *Fancher et al.*, 2010]. For instance, in the last ITRF realization there was available 165 local ties, including the one between the GNSS station and the 14m VLBI radio-telescope at Yebes observatory (no local tie is available so far for the 40m radio-telescope).

There is another means of tying the different techniques known as the "space ties". It is possible to make a combination of different TRFs through the common parameters estimated by each technique as, for instance, the Earth's polar motion. Including these parameters into the combination effectively acts as an additional time-variable co-located point which is free of any tie error, and therefore improves the redundancy of the required tie information. In addition, the space ties can be used to assess the consistency of the available local ties with respect to the space geodesy estimates through the post-fit residuals of the combination. Using this method for the last ITRF combination resulted in only 4% of the available local ties having discrepancies below 3 mm, 42% of them having discrepancies below 6 mm and 37% of them having discrepancies over 1 cm [*Altamimi et al.*, 2011]. These large discrepancies result from the combined effect of systematic errors in the space geodetic techniques and in the local tie measurements. Ideally, as the local ties represent a key element for the ITRF combination, they should be more accurate than, or at least as accurate as, the individual space geodesy TRFs used in the combination. This way local tie errors would not translate into significant internal distortions of the combined ITRF.

The space geodetic techniques are reaching nowadays a high level of internal precision. However, there are still some unmodelled systematic technique-dependent errors that compromise their accuracy. For instance, the multipath and the antenna phase center errors for GNSS stations (the so-called antenna near-field effects; [*Dilssner et al.*, 2008]), and the the radio-telescope deformation for VLBI stations [*Sarti et al.*, 2011]. These technique-dependent errors represent systematic deviations of the radio-electrical points (where station position is measured) from the geodetic reference marker (where station position is referred). Since local ties are determined between the geodetic reference points, these systematic errors also affect the accuracy of local tie measurements. In addition, local ties measured at a single epoch usually contain errors much larger than the internal precision of the space geodetic long-term TRFs, thus increasing the disagreement between both estimates. Therefore, local tie measurements is the critical component of the combination needing further development.

The IERS highly recommend to improve the quality of the local tie determinations in order to perform a better alignment and combination of the VLBI, SLR, DORIS and GNSS TRFs to the 1 mm level overall. For such a goal, local ties should be estimated with an accuracy better than 1 mm in each component. This error budget must consider all systematic and random contributions of each geodetic technique (errors in the coordinates of their reference points) and of the local tie determination itself. For instance, VLBI measurements are usually made with respect to its station geodetic marker. If this point does not exist then the measurements are referred to the radio-telescope invariant reference point (IRP). In any case, the location of the geodetic marker must be determined with respect to the IRP. The IRP is defined as the intersection between the azimuth and elevation axes of the radio-telescope or, if they do not intersect, the right angle projection (the nearest point) from the elevation axis onto the azimuth axis. This point is called "invariant" because, if the axes intersect, the travel time of a radio signal to this point does not depend on which direction of the sky the antenna is looking. However, if the axes does not intersect, then an eccentricity correction needs to be applied to compensate for variation in delay as the antenna points at different elevations. Due to its mathematical definition, the IRP is usually inaccessible. Thus, one of the most difficult parts in determining a local tie comprising a VLBI radio-telescope is the determination and access to its IRP. In addition, local tie surveys involving VLBI stations should include sufficient observations to determine any systematic variation of its reference point due to gravitational and thermal effects. However, typical local ties performed so far, although precise, are just snapshots to survey the surrounding ground control network and the markers of the space geodesy instruments with no regard to variations happening over time, for instance, by temperature fluctuations during the year. These time variations represent a limitation of the accuracy of the estimated local ties which will not be used to combine instantaneous TRFs (at the local tie epoch), but long-term ones (including station positions and velocities). At best, the local tie surveys are done annually. Quite often, the time between surveys is 2-3 years or longer [*Pearlman*, 2008]. Therefore, the best way to provide a local tie determination with improved accuracy is then to monitor continuously its state through time. This implies to automatize the terrestrial survey process without losing the quality of each individual determination. In addition, this approach will allow to identify local station-dependent systematic errors (e.g. monument instabilities, radio-telescope deformation, etc.). This new concept of local tie measurements has begun to take place recently at some geodetic observatories around the world such as Onsala, Wettzell and Goddard [*Lösler and Haas*, 2009; *Neidhardt et al.*, 2010; *Schmeing et al.*, 2010].

3 Local tie at the Yebes Observatory

The Yebes observatory is a research facility of the Instituto Geográfico Nacional (IGN) for technology development in radio-astronomy and geodesy. It is located in the province of Guadalajara, near Madrid, at approximately 930m over sea level. The geographical location of the Yebes Observatory is approximately 40º 31' 29" N latitude and 03º 05' 19" E longitude.

There are currently two co-located space geodetic techniques in Yebes: GNSS and VLBI. The GNSS technique is represented by two stations: YEBE, operating since 1999, has been included in the IGS reference core network (IGS08), and YEB1, operating since 2009. The VLBI technique is represented by two radio-telescopes of 14m and 40m, respectively. The 14m radio-telescope was operational in geodetic VLBI observations from 1995 to 2003 and the 40m radio-telescope is in operation since 2005. In addition, a new radio-telescope of 12m is expected to be installed between 2012 and 2013 in the framework of the VLBI2010 project, and there are also plans to install a SLR station in the future.

The local tie between the GNSS station YEBE and the 14m radio-telescope was measured in 1999. Unfortunately, this local tie was never measured again. In addition, there is no local tie measured so far between YEBE, a newly installed GNSS station called YEB1 and the 40m radio-telescope, nor even between the 14m and the 40m radio-telescopes. This situation clearly needs further improvement. For a proper local tie determination, the IRP of the radio-telescopes at the observatory must be continuously monitored in a stable local coordinate system.

4 Geometric model for the IRP determination

Most of the geometric models typically used in the calculation of the IRP have been based on 3D circle fitting for the positions of markers mounted on the radio-telescope structure under several geometric conditions or constraints [*Dawson et al.*, 2007]. In those models, the radio-telescope is rotated around one axis while the second is held fixed. The observed trajectory of each target located on the radio-telescope corresponds then to a circle. The center points of the adjusted circles for each target are then used to estimate its rotation axis. This process is repeated for many different radio-telescope orientations and for both axes. By minimizing the orthogonal distance between both estimated axes (i.e. the eccentricity) the IRP location is determined. This procedure needs a very long and expensive terrestrial survey which entails three main drawbacks:

- the radio-telescope movements must be planned so that the tracks of the targets form circles in the three dimensional space and thus, a very long observation downtime of the radio-telescope is needed each time the IRP is estimated,
- it cannot be performed in a continuous basis so the monitoring of the IRP location is not possible,

- as the survey probably lasts several days, it probably absorbs sub-daily systematic errors such as thermal deformation of the radio-telescope.

A more efficient approach can be applied if we use the pseudo-random orientations of the radio-telescope during a typical geodetic VLBI observation. With the appropriate geometric model, the IRP can be determined from scattered positions of the targets (not following a planned pattern) using the radio-telescope angle readings as additional observations [*Lösler*, 2008]. This enables the determination of the IRP position while the radio-telescope is performing its inherent observations, and thus, no radio-telescope downtime is expected. Geodetic VLBI observations are preferred over astronomic VLBI observations due to the larger number and variety of orientations of the radio-telescope, but in general all kind of observations could be used to estimate the IRP. In addition, by taking the target observations with an automated tracking instrument (robot tacheometers, laser trackers, etc), the need of a dedicated team of expert surveyors is also reduced and the observations can be taken faster and continuously (even in the darkness). These instruments provide the required high accuracy at a relative low cost (in terms of observation time and manpower).

This new approach is based on a spatial similarity transformation between a coordinate system attached to the radio-telescope (XYZ) and a local coordinate system attached to the observatory ($X'Y'Z'$) without using any constraint on the target positions. The basic assumption of this approach is that points on the radio-telescope rotate around the elevation axis and that the elevation axis itself rotates around the azimuth axis, but no pre-defined geometry nor order of the observations is assumed. Following this new geometric approach (see Fig. 1), any observed geometric vector (O) between an external point (p) with known coordinates and a target (t) attached to the radio-telescope is the sum of three vectors:

- the IRP position (vector X between p and i),
- the eccentricity (vector E between i and v), rotated by the azimuth angle,
- the target position within the radio-telescope system (vector C between v and t), rotated by the elevation angle and by the azimuth angle.

The coordinate system of the radio-telescope (XYZ) is defined being centered in the nearest point of the elevation axis to the azimuthal axis (point v in Fig. 1), the X-axis being parallel to the elevation axis, the Y-axis pointing parallel towards the observed source, and the Z-axis closing the orthogonal cartesian system. For each orientation of the radio-telescope (knowing the azimuth and elevation angles), the position of the targets (point t in Fig. 1) in the coordinate system of the observatory ($X'Y'Z'$) are observed (vector O) from a point (point p) with known coordinates in the observatory system. The vector C, given in the radio-telescope coordinate system, is assumed to be constant and known by construction. However, this vector is actually affected by time-dependent deformations of the radio-telescope structure (e.g. thermal and gravitational effects). These time-dependent deformations of the vector C, unless modeled or observed, will be absorbed by the estimated IRP coordinates (likely propagating into other

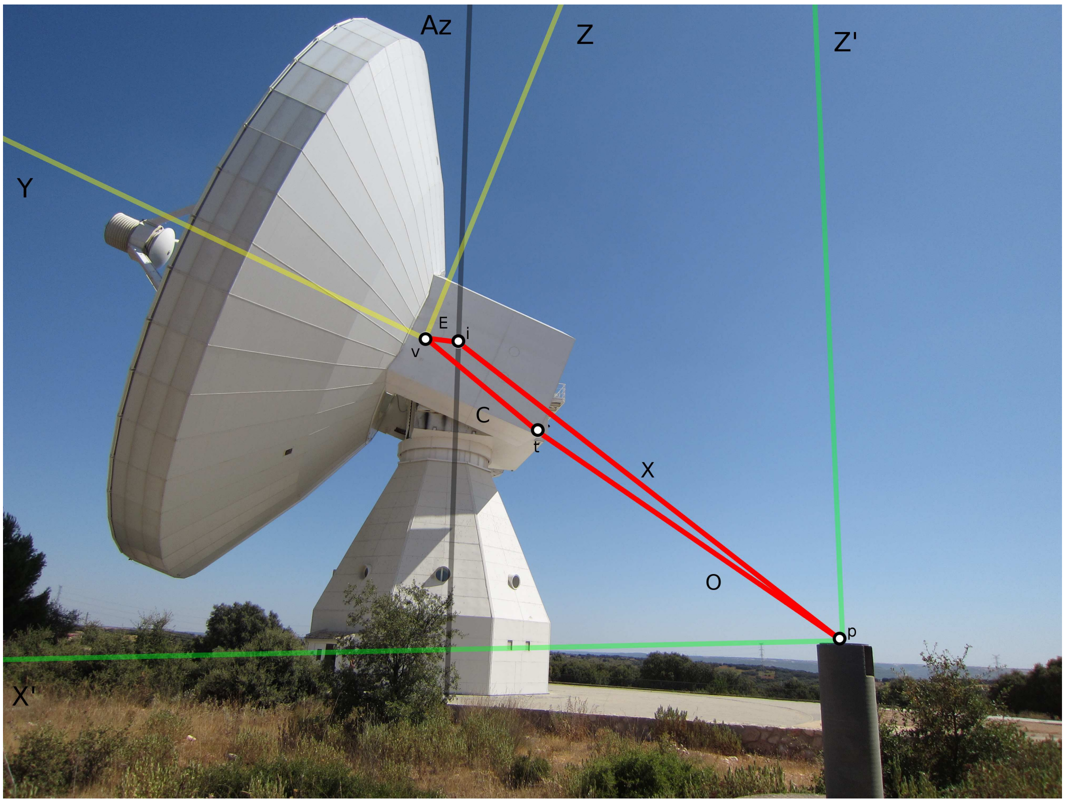

Figure 1: Geometric approach to determine the location of the IRP of the 40m radio-telescope at the Yebes observatory. X', Z': coordinate system of the observatory; Y, Z: coordinate system of the radio-telescope; Az: radio-telescope azimuth axis; p: observing point; t: observed target; v: radio-telescope elevation axis; i: radio-telescope IRP; O: observed vector; C: constant vector; E: eccentricity vector; X: unknown vector.

estimated parameters too) in the form of systematic biases and increased uncertainty. However, since the deformation will translate mostly in a 1:1 bias of the IRP coordinates, this geometric model can also be used to monitor such deformations through the variations of the IRP coordinates. Thus, this new method allows to derive essential information about the temporal behavior of the radio-telescope that could be used by the analysis centers processing VLBI data in order to improve the station position and other estimated parameters (e.g. Earth rotation, tropospheric delay). This is one of the major advantages of this methodology. By integrating successive short observation spans, this approach can be used to asses the impact of time-variable deformations on the IRP coordinates [e.g. *Neidhardt et al.*, 2010], although the uncertainties of the estimated parameters will be increased. For the simulations carried out here and for the scope of this study we assume here that the vector C is constant. Nevertheless, very short observations spans will be simulated to assess the suitability of this method to estimate the short-term deformations of the radio-telescope.

Observing the value of vector O at different epochs (e) and knowing the value of vector C, the remaining unknown parameters to estimate are the vector X, representing the position of the IRP (i) in the local coordinate system, and the eccentricity (E) in the radio-telescope coordinate system. The geometric model can be written this way:

$$X + (E + C)R_e = O_e \tag{1}$$

where R is a rotation matrix formed by the azimuth and elevation angles of the radio-telescope at each epoch e. Equation 1 can be simplified into:

$$X + M_e = O_e \tag{2}$$

being M the transformation matrix between at each epoch e between the local and the radio-telescope coordinate systems. Since a perfectly constructed radio-telescope does not exist, to estimate a reliable IRP, the transformation matrix (M) has to allow for some extra unknown parameters relating both coordinate systems. These parameters correspond with the vertical deviation of the radio-telescope with respect to the local coordinate system (difference between Az and Z' in Fig. 1) and also the non-orthogonality between the azimuth and elevation axes (not shown in Fig. 1). Both parameters can also be adjusted together with the IRP coordinates and the eccentricity. This way, the matrix M is defined as:

$$M_e = R_y(\alpha)R_x(\beta)R_z(\theta_e)R_y(\gamma)(E + R_x(\varepsilon_e)C) \tag{3}$$

where:
α is the vertical deviation of the radio-telescope around the Y axis of the observatory system,
β is the vertical deviation of the radio-telescope around the X axis of the observatory system,
θ is the azimuth angle of the radio-telescope around its primary (Az) axis at epoch e,
γ is the non-orthogonality angle between X and Z axes around the Y axis,
ε is the elevation angle of the radio-telescope around its X axis at epoch e.

Equation 2 can be re-written as the following system:

$$F(x) = L \tag{4}$$

where $F(x)$ is the functional model relating the parameters to estimate (x) with the input observations (L). We can estimate from a minimum of 3 parameters (coordinates of the IRP in the local coordinate system) to a maximum of 7 (adding the eccentricity E, the non-orthogonality of the axes γ and the vertical deviation of the radio-telescope projected in the two horizontal components α and β). Solving this system corresponds to solve a non-linear weighted least-squares fit. Thus, the linearization of equation 4 is performced through the first term of a Taylor expansion around an a priori value of the parameters to estimate (x_0). This results in:

$$F(x_0) + \frac{\delta F(x)}{\delta x}|_{(x=x_0)}(x - x_0) = L \tag{5}$$

Equation 5 can be simplified in the following linear equation system:

$$AX = T \tag{6}$$

where:
$A = \frac{\delta F(x)}{\delta x}|_{(x=x_0)}$ is the Jacobian matrix of partial derivatives with respect to the parameters.
$X = x - x_0$ is the vector of estimated corrections to the a priori values of the parameters,
$T = L - F(x_0)$ is the vector of residuals, that is, the observed minus computed observations.

Extracting the parameters to estimate from equation 6 and weighting the input observations results in:

$$X = (A^t W A)^{-1} A^t W T \tag{7}$$

where W is the stochastic model of the observations (weights). Without any a priori information about the stochastic model, we assume that the field observations of the targets are uncorrelated (following a white noise model). The W matrix becomes then diagonal where each element corresponds to the inverse of the variance of each observation.

The solution of the system of equation 7 can be obtained by iteration until a minimum correction of the a priori values is reached. Assuming uncorrelated field observations, the formal precision of the estimated parameters is a function of the number, precision and geometry of the observations. These three conditions will be simulated and tested in the next section. However, the assumption of uncorrelated field observations is actually unrealistic due to mismodelled/unmodelled systematic errors as, for instance, tropospheric delay, monument expansion, etc. The resulting formal uncertainties of the estimated parameters will be then more optimistic than realistic. Further development of the stochastic model of the observations is needed in order to obtain more realistic uncertainties. A different, and probably more simple, procedure would be to properly scale (inflate) the estimated variance factor of the estimated parameters. For the simulations carried out in this study, the uncorrelated field assumption is however valid since we aim at comparing relatively the performance of the IRP determination under several observation conditions. Thus, departures of the white noise model for the observations can be neglected as they are assumed to be similar in all the simulated scenarios.

5 Simulation of the IRP determination

Simulation studies are recommended to be carried out before performing the local tie survey in order to tune the field observation strategy [*Kallio and Poutanen*, 2010]. To perform the simulation of the IRP determination of the 40m radio-telescope at the Yebes observatory we need three different set of numerical values.

First, we assigned predefined values to the parameters to be estimated later. Since we are aiming at the accuracy (biases) and precision (repeatability) of the estimated parameters following different observation approaches, the true value of these parameters is of minor importance. This way the values for the coordinates of the IRP in the local coordinate system, the eccentricity, the non-orthogonality of the axes and the vertical deviation of the radio-telescope were fixed to zero.

Second, we assigned the number of targets and their coordinates in the radio-telescope system (vector C). The number of targets is limited by the fact that, without an automatic target identification system, the surveying instrument (e.g. a robot tacheometer) would be able to track only one target at a time for each position of the radio-telescope. The maximum number of targets are then limited by the acceptance angle of the target retro-reflectors, i.e, the angle within they can be observed. From the large variety of different possibilities to set the targets, we chose to simulate two targets on the external face of both counterweights. In addition, to increase the acceptance angle, each one of these targets is simulated to be composed indeed of four mini-corner-cube retro-reflectors (acceptance angle of $90°$ each) allowing thus for a joint hemispherical acceptance angle. This way, we ensure that one and only one of the targets, unless being both hidden by the radio-telescope parabola, will be observed at any time from any place. We neglect in the simulation the possibility that more than one corner-cube could be observed, likely resulting in a range error. This issue should be addressed before the local tie survey by calibration of the instrumentation. The coordinates of both targets in the radio-telescope system, obtained by construction design, are (note the different sign of the X coordinate):

- $X_{counterweight\,left} = -6.82m$ $X_{counterweight\,right} = 6.82m$
- $Y_{counterweight\,left} = -5.29m$ $Y_{counterweight\,right} = -5.29m$
- $Z_{counterweight\,left} = -4.88m$ $Z_{counterweight\,right} = -4.88m$

In addition, in order to simulate errors on the true location of the targets, for each simulated experiment we added systematic errors to these coordinates of the same amplitude as the random observation errors (see below).

Third, to simulate realistic radio-telescope orientations we used a set of real elevation and azimuth values extracted from the program schedule of a geodetic VLBI observation of the 40m radio-telescope. This resulted in 413 different values of azimuth and elevation. Possible errors of the radio-telescope orientation angles were not considered in this simulation but should be taken into account in the future.

Using all these values (parameters of the radio-telescope, coordinates of the target positions, and orientations angles) the simulation of the field observations using equation 2 is straightforward. A perl script (*lt_sim.plx*) was written to perform the simulation of the observed targets automatically. Due to the relative location of the instruments and targets, we also took into account in the simulation the unobservable targets hidden behind the dish. That is, knowing the location of the observing instrument and the size of the radio-telescope, zero or one target will be observed from each instrument depending on the azimuth/elevation of the radio-telescope.

The coordinates estimated from the geometric model of equation 2 contain no observation errors. Therefore, we added to the estimated target coordinates a white noise variance simulating the random uncertainty of the observed target coordinates. A vertical constant offset simulating elevation-dependent deformation of the counterweights due to gravitational effects were also added to the target coordinates in first simulations, but they were removed since it resulted in a 1:1 propagated vertical bias of the IRP coordinates. More detailed information about the a priori gravitational deformation of the counterweights will be derived in the future from a finite element model of the radio-telescope.

The next step was to introduce these simulated observations into the mathematical model described in section 2 and to re-estimate the simulated parameters through a least-squares adjustment. The same uncertainty was assigned to all the simulated observations (unity weight matrix) in order to simulate blunders in the observations, which were identified and removed iteratively from the estimation process. A second perl script (*lt_solve.plx*) was written to perform the least-squares fit.

The final step was to repeat this process (simulating the observations with white noise variance and estimating the parameters) 1000 times using the Monte Carlo method for each different configuration. Using such a large number of simulations allows us, through the central limit theorem, to evaluate the accuracy (mean bias) and precision (dispersion) of the estimated parameters for each configuration tested.

Following this procedure, we analyzed by simulation the accuracy/precision of the estimated parameters to decide on:

- the number of observing instruments needed:
  the best geometry of the observations would allow to always observe the targets for every orientation of the radio-telescope, that is, with a range of $360°$ in azimuth and $90°$ in elevation. This could be accomplished by setting a large number of surveying instruments. However, it is desirable to use the minimum number of surveying instruments as possible. Otherwise, the (unknown) geometric vectors between the different surveying instruments will introduce some additional bias and larger uncertainties on the local coordinate system (not to mention the unnecessary increment of costs). We simulated one, two and three observing instruments separated by $120°$ in azimuth from the radio-telescope. The distance of the instruments to the radio-telescope was not tested since we simulated directly the observed target coordinates. Nevertheless, a broad range in the precision of the observed targets was simulated (see next point) allowing to consider this issue in the future (e.g. assessing the precision of the observations with distance).

- the accuracy needed in the measured target coordinates:
  the accuracy of the target coordinates depends on the accuracy of the field observations (quality of the measuring instrument, tropospheric corrections, distance, inclination, etc.),

but also on the accuracy of the local coordinate system, i.e., the coordinates of the observing instrument(s). Using only one observing instrument would allow to cancel any bias of the local coordinate system as it will be realized always by the same physical point. Even if this point moves with respect to the radio-telescope (inclination, expansion) the local tie between the GNSS and the VLBI stations will remain unaltered, as long as both stations are observed simultaneously. That is, in theory, we could select each time a different point to realize the local coordinate system and the geometric vector tying the GNSS and VLBI reference points will be always the same. However, in practice, the local coordinate system must be usually permanent and available (observable) for/from several co-located techniques. This implies to realize the local coordinate system by means of several reference points which coordinates are determined continuously. These coordinates contain however systematic errors and uncertainties that must be properly propagated to the final local tie variance-covariance matrix. To simulate a realistic error budget comprising both sources (measurements and local coordinate system) several measurement noise levels were considered in the simulation, from 3 to 15 mm (1 sigma). A more realistic case to be analyzed by simulation in the future will consist in adding systematic biases to the coordinates of the observing instruments in addition to the observing random errors.

- the minimum number of observations needed:
  large observation time spans (e.g., 24h) will allow to determine the IRP coordinates with high precision. However, any unmodelled systematic phenomenon affecting the IRP location at shorter time scales (e.g., thermal and gravitational deformation) will be also integrated by the least-squares adjustment affecting thus the estimated IRP coordinates and other parameters. Inversely, shorter observation time spans may be selected to directly observe those unmodelled systematic effects in order to monitor the performance of the radio-telescope or to be able to model them in the future. In this simulation, we considered several observation time spans from 1 hour to 24 hours. These observation time spans are relative to the number of different radio-telescope orientations used in the simulation (413). This should be taken into account if bigger of smaller experiments (in number of orientations) are going to be used to estimate the IRP coordinates.

6 Results

In this section we show the numerical values resulting from the comparison between the predefined and the estimated parameters after reducing the simulated target coordinates under several observing scenarios, that is, different observation span, varying observing instruments and different quality of the observed target coordinates. By estimating the mean value for the 1000 repetitions of each scenario, we can assess the biases (accuracy) of such parameter in different observation schemes. Likewise, by estimating the dispersion around the mean value, we can assess the precision of the estimated parameters for each simulated scenario. The requisites of the necessary number of observations, the observing geometry, and the precision of the observations for each parameter will be assessed. As mentioned in Section 5, the errors of the observed target coordinates depend mainly on the observed distance and the quality of the observatory coordinate system. The precision (dispersion or repeatability) of the seven estimated parameters for all the simulated scenarios is represented in Figs. 2, 3, and 4 of the Appendix.

With respect to the IRP coordinates (Xr, Yr, and Zr), the maximum bias reached 0.4 mm for the scenarios with larger noise amplitudes (> 10 mm), shorter observation spans (< 3 h) and worse observation geometry (only 1 instrument). Improving the observation precision (below 6 mm), improving the observation geometry (using at least 2 instruments) or increasing the observation period (more than 4h) reduces to minimum (< 0.2 mm) the biases of the IRP coordinates. This demonstrates that the geometric model is unbiased and that, with a large number of observations, the introduced random errors are properly averaged out. On the other hand, Table 1 shows the precision of the field observations needed in order to keep the precision or the repeatability of the estimated IRP coordinates below 1 mm for each component. This table confirms that with an observation period of 24h the IRP coordinates will be confidently estimated even with only one observing instrument. However, it is impossible to estimate the IRP coordinates with one instrument using very short observation periods, and that even with larger observing periods (12h), one instrument will be likely not enough. Conversely, the IRP coordinates could be estimated using 2 instruments during 4h, or 3 instruments during 3h. From these simulations, it seems that little improvement is achieved from 2 to 3 instruments for the IRP determination.

	24h	12h	6h	4h	3h	2h	1h
1 instrument	12 mm	3 mm	3 mm	3 mm	NA	NA	NA
2 instrument	15 mm	9 mm	6 mm	6 mm	3 mm	3 mm	3 mm
3 instrument	15 mm	12 mm	9 mm	6 mm	6 mm	3 mm	3 mm

Table 1: Estimated observation precision needed to reach 1 mm in all components of the IRP (Xr, Yr, and Zr) using different number of observing instruments (left) and different observation lengths (top). NA means that this precision is not reached.

The eccentricity parameter (Ec) showed maximum biases at the 0.1 mm level for the all the simulations. In addition, the precision of the estimated eccentricity below the 1 mm level is easily reached even when using only one observing instrument and an observation period of 3h (see Table 2). This shows that the eccentricity is mostly insensitive to the precision and number of the observations (number of instruments and observation time span). The estimation of this parameter is highly precise since it only depends on the azimuthal distribution of the observations (2D circle fitting), which is mostly ensured in geodetic VLBI observations of a few hours.

The vertical deviation or tilt of the radio-telescope (Vx, Vy) with respect to the observatory coordinate system is heavily affected by the position of the targets and the geometry of the observation. The farther from the rotation axes are located the targets, the more accuracy and precision is attained for this parameter. For the location of the simulated targets (on both counterweights), biases of some arc-seconds (< 3) were found using only one observing instrument, even for 12h and 24h observation periods. Using two or three observing instruments, these

	24h	12h	6h	4h	3h	2h	1h
1 instrument	15 mm	15 mm	9 mm	6 mm	6 mm	3 mm	3 mm
2 instrument	15 mm	15 mm	12 mm	9 mm	9 mm	6 mm	3 mm
3 instrument	15 mm	15 mm	15 mm	12 mm	12 mm	9 mm	6 mm

Table 2: Estimated observation precision needed to reach 1 mm in the eccentricity (Ec) using different number of observing instruments (left) and different observation lengths (top).

biases were significantly reduced to the arc-second level. Apart from these possible biases, a precision of 10 arc-seconds (equivalent to 1 mm of the IRP coordinates) is only confidently obtained when observing more than 12h with two or three instruments (Table 3). This is a relative low precision taking into account that the tilt of the azimuth axis has been estimated with inclinometers to be near 10 arc-seconds [*de Vicente*, 2010]. The precision of the tilt improves below 5 arc-seconds only when observing 24h with three instruments. This is probably related to the inadequate relative position of the targets with respect to the rotation axes. Placing the targets far from the rotation axes, i.e. on the parabola, would allow (from a mathematical point of view) a better determination of the tilt. However, larger time-dependent deformations should be then considered (modeled) in order to avoid biases on the estimated parameters. Therefore, based on these simulations, the tilt of the 40 radio-telescope could not be confidently solved following this procedure. This parameter should be estimated with an independent approach as, for instance, the inclinometers actually installed the encoders of the elevation axis [*de Vicente*, 2010] and then used as additional observations to estimate the IRP coordinates.

	24h	12h	6h	4h	3h	2h	1h
1 instrument	3 mm	NA	NA	NA	NA	NA	NA
2 instrument	6 mm	3 mm	3 mm	NA	NA	NA	NA
3 instrument	6 mm	3 mm	3 mm	3 mm	3 mm	NA	NA

Table 3: Estimated observation precision needed to reach 10 arc-seconds in the tilt (Vx, Vy) using different number of observing instruments (left) and different observation lengths (top). NA means that this precision is not reached.

With respect to the non-orthogonality parameter (Dl), we found biases at the 1 arc-second level when observing very short periods (< 6h) using one, two or three observing instruments. These biases were almost completely reduced with at least 12h of observation. As shown in Table 4, a precision better than 10 arc-seconds can be obtained when observing more than 4-6h with one, two or three instruments. From this table it seems that there is little improvement in the estimation of the non-orthogonality of the axes when increasing the number of observing instruments. If the field observations are at the 3-6 mm level, a precision of 5 arc-seconds can be obtained in the non-orthogonality observing 12h with one instrument, 6h with two instruments, and 4h with three instruments.

	24h	12h	6h	4h	3h	2h	1h
1 instrument	12 mm	6 mm	3 mm	3 mm	3 mm	NA	NA
2 instrument	12 mm	9 mm	6 mm	3 mm	3 mm	3 mm	NA
3 instrument	15 mm	9 mm	6 mm	6 mm	3 mm	3 mm	NA

Table 4: Estimated observation precision needed to reach 10 arc-seconds in the non-orthogonality (Dl) using different number of observing instruments (left) and different observation lengths (top). NA means that this precision is not reached.

7 Conclusions

The motivation for the simulation was to assess to what extent an automatic system based on robotic total stations could be properly configured to continuously monitor the invariant reference point of the 40m radio-telescope at the Yebes observatory. The analysis carried out with simulated data has proven that the geometric model used is a successful improvement in the IRP determination of VLBI radio-telescopes with respect to the classical 3D circle fitting. This method, in combination with modern surveying instruments, represents a precise, automatic and continuous estimation procedure of the IRP coordinates, while avoiding expensive manpower and downtime of the radio-telescope.

It has been shown that, with an appropriate observing geometry (including several observing instruments), this approach is able not only to accurately estimate the IRP coordinates and other parameters (e.g. eccentricity and non-orthogonality of the axes), but also it allows to monitor its temporal behavior, thus providing fundamental information about the radio-telescope deformation due to gravitational, thermal, wind, etc. effects.

Based on these simulations, we conclude that only one robotic total station would allow to reliably estimate the IRP coordinates in 24h observation batches, provided that the precision of the observed target coordinates is better than 1 cm (to be tested in the future with real measurements). In order to monitor short-term variations of the IRP coordinates, mainly due to deformation of the radio-telescope, two instruments could be used in 3-4h observation batches, or three instruments in 2-3h observation batches. These observation periods could be reduced further if the accuracy of the field observations (comprising the accuracy of the observatory coordinate system) is improved bellow the 3 mm level, which at this moment seems not very feasible due to unmodeled or mismodeled errors (e.g., tropospheric delay, uncalibrated equipment, instrument quality, monument expansion, model deficiencies, etc).

A new IRP monitoring system, based on the presented approach, will start up in the upcoming months for the 40m radio-telescope at the Yebes observatory.

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9 Appendix: Figures

9.1 One observing instrument

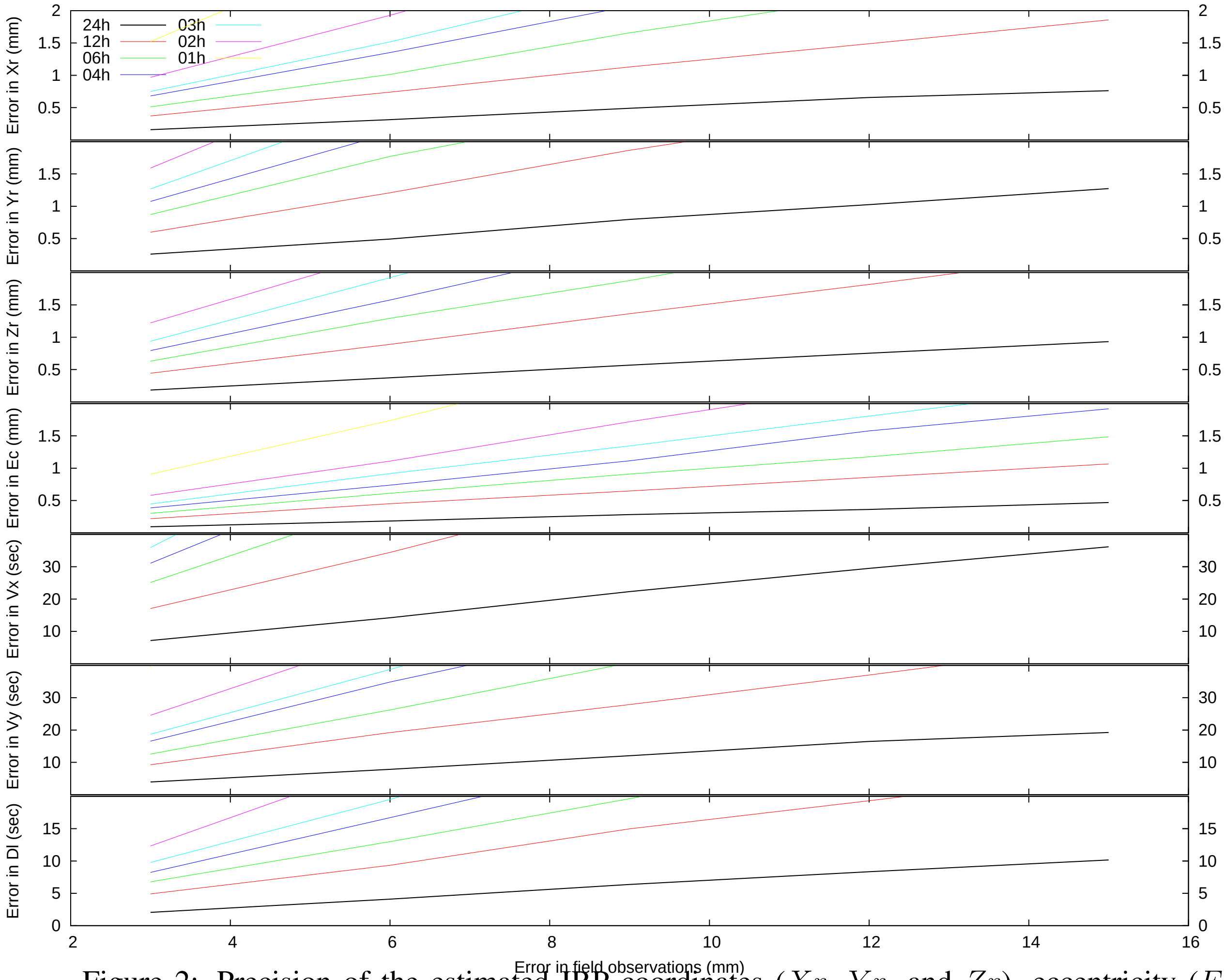

Figure 2: Precision of the estimated IRP coordinates (Xr, Yr, and Zr), eccentricity (Ec), vertical deviation (Vx and Vy), and non-orthogonality (Dl) parameters estimated with one observing instrument and different precision of field observations

9.2 Two observing instruments

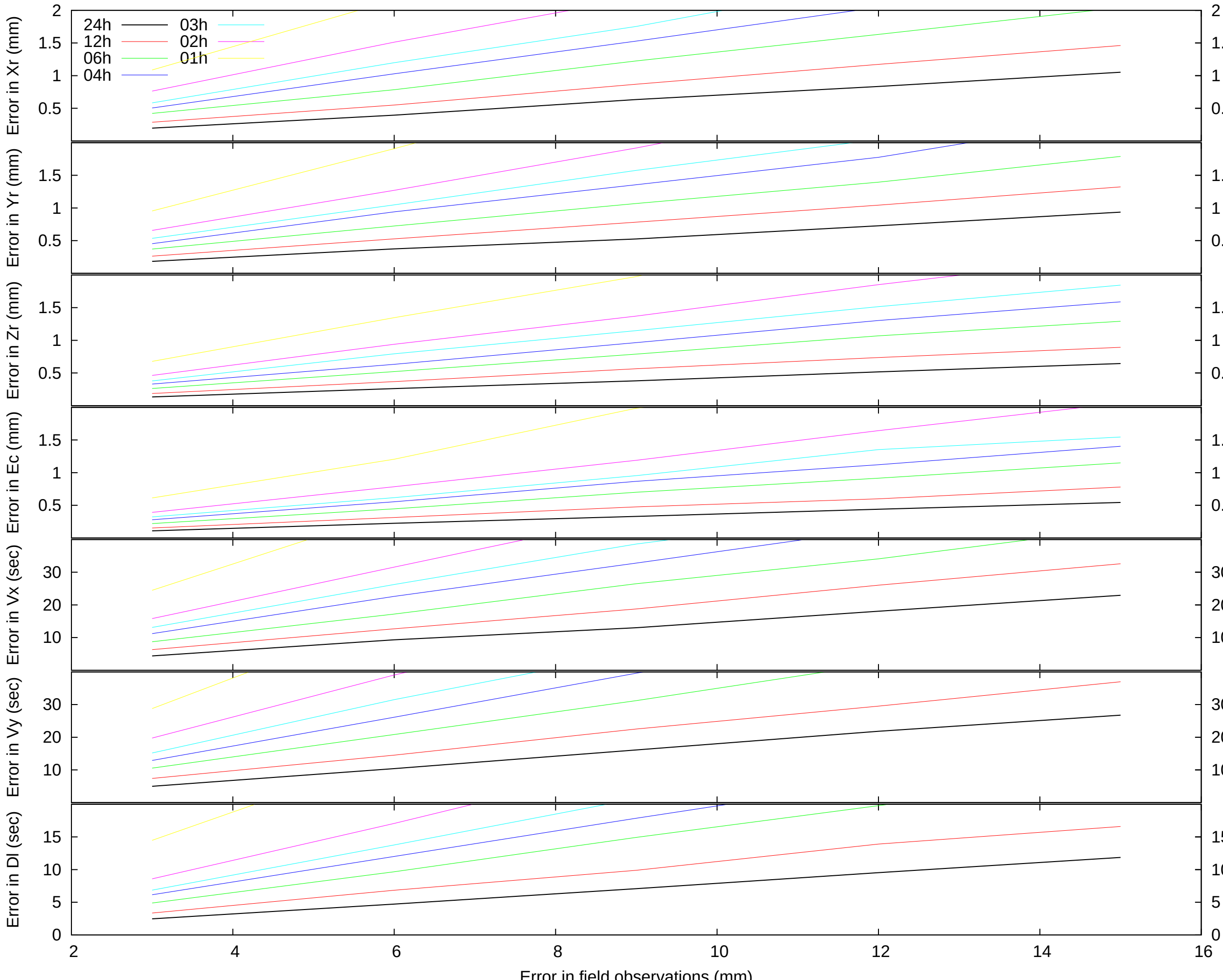

Figure 3: Precision of the estimated IRP coordinates (Xr, Yr, and Zr), eccentricity (Ec), vertical deviation (Vx and Vy), and non-orthogonality (Dl) parameters estimated with two observing instruments and different precision of field observations

9.3 Three observing instruments

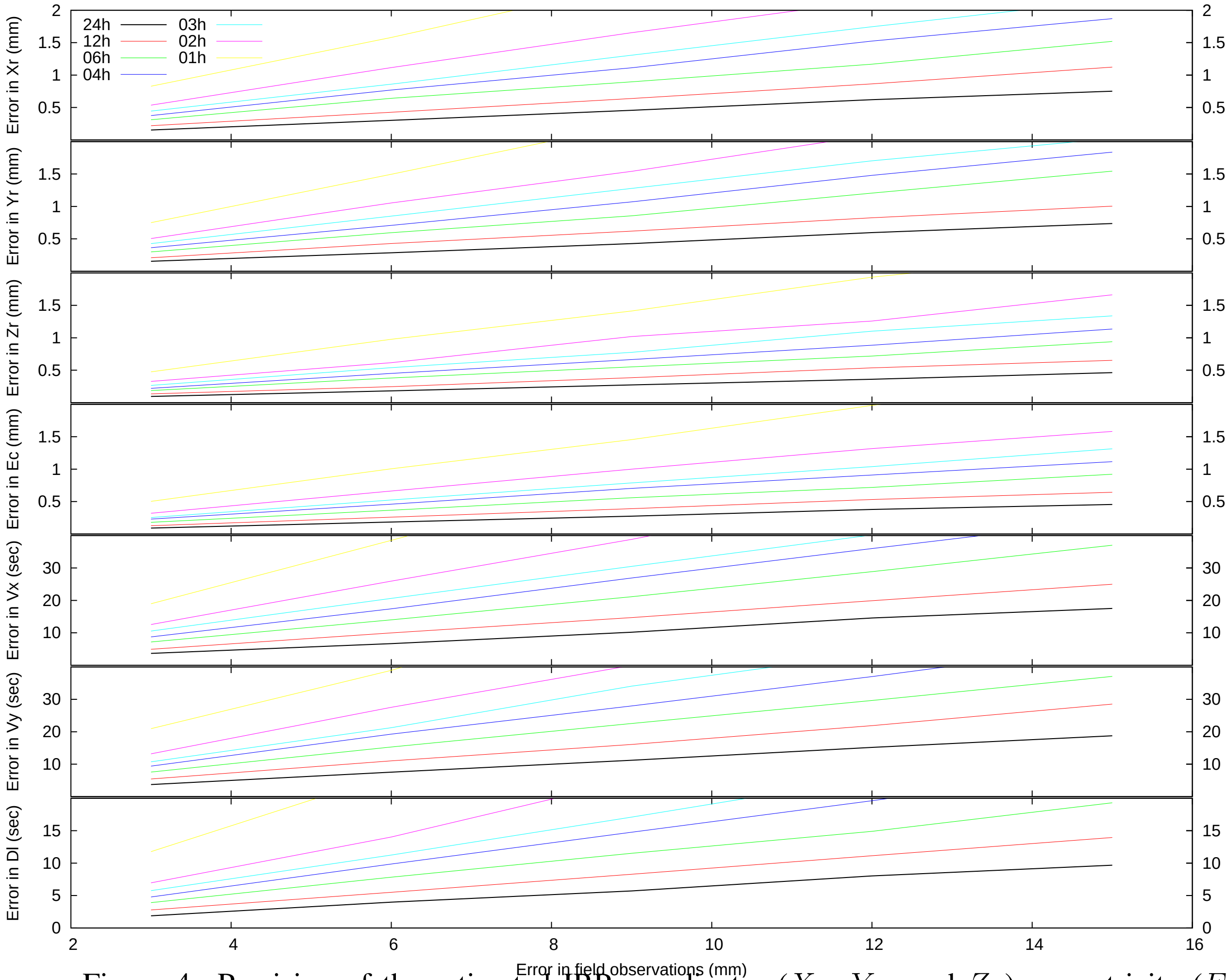

Figure 4: Precision of the estimated IRP coordinates (Xr, Yr, and Zr), eccentricity (Ec), vertical deviation (Vx and Vy), and non-orthogonality (Dl) parameters estimated with three observing instruments and different precision of field observations